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1824
563

LINEAL DRAWING,

AND

INTRODUCTION TO GEOMETRY,

AS TAUGHT IN THE

Lancastrian Schools of France.

LINEAL DRAWING,

AND

INTRODUCTION TO GEOMETRY,

&c. &c.

J. M'Creery, Tooks-court.
Chancery-lane, London.

1.H. 1825

LINEAL DRAWING,

AND

INTRODUCTION TO GEOMETRY,

AS TAUGHT IN THE

Lancastrian Schools of France;

TRANSLATED FROM THE FRENCH

OF

MONSIEUR L. B. FRANCŒUR,

ONE OF THE SECRETARIES OF THE SOCIETY OF EDUCATION,
PROFESSOR OF THE ACADEMY OF SCIENCE, &c. &c.
AT PARIS.

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1824.

563

TRANSLATOR'S PREFACE.

The cyphers or measures made use of in this work, are merely signs employed to convey ideas of linear proportions, and a few simple Geometrical Problems.

The Translator of the following pages has been induced to adhere to the French standard of measure;

1st, From the wish, that in all the material parts of the work, the translation should be as literal as possible.

2dly, Because the French standard of measure, from its uniformity, its simplicity, and the facility with which all its operations are performed by means of decimal numbers, affords a vehicle of instruction which offers many advantages. The attention of the pupil is not distracted by a complicated arithmetic, from the pursuit in which he is engaged, and which being a distinct study from arithmetic, should not have its difficulties increased by a tedious process of calculation.

3dly, The art, which it is the object of this work to teach, being once acquired, and the eye once formed to accuracy, it will adapt itself with equal facility to whatever standard of measure

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Hectogramme = 3 oz. 8½ drachms, Avoirdupois.
Kilogramme = 2 lb., 3 oz., 5 drachms, Avoirdupois.
Myriogramme = 22 lb., 10 oz., 15 drachms, Avoirdupois.
1 Quintal Metrique, or 10 Myriog. = 220 lb., 10 oz., 1 dr.
10 Quin. Met., or Millimetre, = 1 ton less, 33½ lb.

To find the exact length of the metre, from which all the other measures are deduced, find the length of a pendulum, which shall oscillate sixty times in a minute.

ELEMENTS OF DRAWING,

&c.

INTRODUCTION.

THE art of Lineal Drawing, the faculty of estimating proportions by the eye alone, and of tracing them on paper, would be a useful acquirement in most of the occupations of the middling and lower classes of society, but its direct advantages are perhaps not so important as the *habit* it encourages of accuracy and precision, which, when acquired in youth, in any one instance, usually extends its influence over the general character. The art of Lineal Drawing may not contribute as much as the power of reading and writing, to improve the moral sense; but it is perhaps more conducive than these to the prosperity of industry, to the practical success of each individual in his trade. It was first introduced by the Duke de Caeyes, in his School for the Poor, at Livourne; and the experiment was so satisfactory in its result, both as to the facility with which the art was acquired by the pupils, and the advantages attending it, that it has since become a branch of instruction in most of the Lancastrian schools for the education of the poor throughout France.

GENERAL INSTRUCTIONS.

It is not expected that there should be any previous knowledge of drawing possessed by either the master, the monitors, or the pupils; but that each, equally ignorant both of Geometry and of Drawing, shall succeed in accurately tracing all imitation of forms by mutual instruction, and the force of example and imitation.

The master should, in the first instance, study the general rules laid down in this work, so as thoroughly to comprehend them; he must then begin to form monitors, choosing six or eight boys from the best of the writing class, acting as their monitor, and exercising them in the rules to be explained. When these are fully acquainted with the method, and are perfect in the rules of the first class, the instruction may begin in the school. Classes of design being formed from the three last classes of writers, and one of the best of the monitors being appointed to each class. The master should still continue to devote a portion of time, after school hours, to the instruction of his monitors, till they comprehend the whole of the practice, and the rules which they are to dictate, and thus, by degrees, the six classes of design will be introduced into the school. The monitors must be furnished with several kinds of instruments. There must be provided for the drawing classes:

1st, Slates.

2dly, Black Tablets of wood,* at least one metre long, by seven decimetres high,† the children are to draw on these with white chalk. These boards are fixed to the wall, in the part of the school-room allotted to the half circles of the reading classes, and at a height which the children may draw on conveniently.

3dly, Wooden or Pasteboard Tablets, on which should be pasted separately the sheets of the engraved diagrams, which are subjoined to this work, and which should be suspended above the tablets the children draw on, being the models they should copy.

4thly, Demimetres,‡ divided into decimetres and centimetres, in the form of rulers, with brass or iron ends; these answer the purpose of monitors pointing sticks. This measure should also be nailed or painted on each tablet, that the child may have it constantly before his eye.

5thly, Small Rulers of two or three decimetres,§ divided into centimetres and millemetres. These the monitors will keep in drawers for occasional use.

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* These tablets are made of oak plank, rubbed with lamp black mixed with glue, and occasionally afterwards with the sediment of the ink bottles, which keeps them of a good colour.

† 3 feet 4 inches, by 30 inches.

‡ Lines of about 20 inches.—See Preface.

§ Lines of 10 inches.

6thly, Small Pasteboard Tablets, on which are pasted the series of orders which the monitors are to give, and which are affixed at the end of this work: these the monitors hold in their hand, and dictate from.

7thly, Large and small Squares, which are given to the monitors to verify if the angles drawn are rectangular; the large squares are for the black tablets, the small squares for slate exercises.

8thly, Large and small Compasses of wood or iron.

9thly, Lastly, Protractors, or graduated semicircles.

The instruction of drawing is conveyed exactly like that of writing and arithmetic. In the semicircles before the black tablet, the monitor reads an order from his tablet, the class execute it, seeing the model they are to imitate, and the correction is made on the spot by another child. In the exercises on the slates, the class draw without a model, and the monitor corrects without speaking.

It is not only necessary to exercise the hand; the eye should also acquire precision in the estimation of distances, the direction of lines, and the form of outlines. There is as much merit in correcting a stroke, or even in perceiving in what part, and where it is defective, as in tracing it correctly. The monitor, therefore, learns as much as those he is teaching, and improves himself, while he is instructing others.

The children in the classes never are allowed to use ruler, square, or compass; and must acquire by habit, the power of tracing the figures required with accuracy. These instruments are in the hands of monitors only, who use them as tests. It has been ascertained, that unless a child is deprived of all talent whatever, he will soon be able to draw straight lines, circles, ellipses, and the different combinations of these figures, with great correctness. A pupil must never be permitted to turn his slate, but must attain a facility in drawing his lines freely, in every direction, while his slate remains in the same position. He must accustom himself to calculate specified measures with his eye, both in lines and cubes. Practice is sufficient to render the sense of sight an infallible guide in the measure of space and distances; and graduated scales of decimetres, centimetres, and millemetres, affixed to the tablets, and thus perpetually before his eye, will contribute to it.

The pupils for lineal drawing, should be chosen from the three last class of writers, and these should form six classes for drawing. Writing being a species of drawing, comprehending even more complicated forms than those of our first classes, it might appear eligible to allow the boys to draw simple lines, previous to their attempting to write; but as the practice of writing is conducive to improvement, both in reading and orthography, which are essential objects not to be lost sight of, it becomes expedient to defer the instruction of drawing, till the children have made some proficiency in writing.

There are a multitude of words, such as diameter, parallel, rectangle, &c., used in Geometry, which have precise acceptations; these the master should be acquainted with, and we shall explain them as we proceed: but it is to the first monitors only, that it may be useful to convey these instructions; the other pupils have no occasion for them. The form of the models, and the habit of imitating them, is sufficient to affix to these words their precise meaning, without any explanation being necessary.

The classes being formed, and the monitors assigned to each, the children are taught either in semi-circles, before the tablets affixed to the wall, or seated on benches.

1st, *The Scholars ranged in semi-circles.*

Each pupil is furnished with a piece of chalk, not pointed; above the black tablet is placed the board, on which is pasted the sheet of engraved figures which he is to copy, and which are seen by the whole half circle. The monitor stands against the wall, holding a half metre measure as a pointing-stick. He points to one of the class, who advances, and draws the diagram required. If the figure is tolerably well executed, or that there is no reason to suppose that the next would do it better, it is rubbed out, and the boy, No. 1, returns to his place. No. 2 draws the same figure, if it is defective, the monitor says "*next*;" No. 3 advances to correct the drawing; but if he places his crayon so awkwardly, that he shews he does not perceive the fault, the monitor again says "*next*," and No. 3 returns to his place; No. 4 advances, if he succeeds tolerably in the correction, the word "*pass*," tells him to take the place of No. 2, having first rubbed out the figure. When all the boys in the class have drawn the same diagram, the monitor must, in his turn, draw it accurately, by the ruler, or the square. He then rubs out his own drawing, and proceeds to the next diagram. A second figure is never to be formed, till the first is rubbed out.

At a given signal, the monitor turns the tablet of engraved diagrams to conceal the model, and the class draw from memory, according to the same order of dictation they did before; after which, they return to their seats, to trace again the same figures on their slates, and without a model.

2dly. *The Children seated on their Benches.*

In the same manner as in the reading classes, where the children write the words which they had read in the semicircles; they should now repeat on the slates, the figures they had previously traced on the black tablet. A signal is given by the monitor when his class have filled their slates, when all the classes have finished, the monitor general gives the order to correct: each monitor then proceeds to the correction without speaking. To half the class he applies the rule or square to shew the defects; on the slates of the other half he traces the figures more correctly, to shew the irregularities; these corrections must be made with accuracy and rapidity, and should not consume more than four minutes of time.

Each monitor dictates in his turn, as in the writing classes. When the general monitor gives orders to *begin*, the monitor of the fifth class immediately dictates; then that of the fourth; and so on to the last; then the monitor of the fifth class receives a new order to begin. In his class some of the diagrams will require a longer time to draw than the others, in which case he does not dictate, but passes the word to the following class, and waits to dictate till his next turn. The children of his class will in this case be required to draw but three or four figures, while the others draw six.

The monitor should generally dictate, that the diagrams be drawn of specified lengths, which he must frequently vary; thus for instance, he reads on his tablet, *trace a square*, he may add *of three centimetres*, *five centimetres*, &c. &c.

It will frequently happen that *two* pupils will have failed in different parts of their drawing: one for instance will have traced a crooked line, but the right line intersecting it, will be perfectly perpendicular; the lines of the other will be right lines, but not perpendicular: it may be embarrassing to know which should have the priority. But the monitor ought to give the preference to him who has accomplished the thing most difficult; and the degree of difficulty is marked by the rank of the questions on his tablet, which are ranged accordingly. Therefore, in our example, the first pupil should have the preference over the other.

Generally speaking, the monitor should dictate two different diagrams, each repeated three times; and these six diagrams should fill the slate. When all the slates are filled, and the order for correction is given, the monitor passes in review *all* the slates; he then corrects the slate of every alternate boy as he ascends the class; on his return, he draws the two diagrams accurately on those alternate slates he did not correct. These corrections are rapidly made by means of his rule and square; the very defective figures he draws a line across. The size of the class should be regulated by the number of slates which the monitor can correct in four minutes.

It is better not to wait to go to a new figure, till the boys are perfect in tracing the preceding one; it is sufficient that they should do it tolerably well; for they should not be fatigued or disgusted by requiring them to repeat perpetually the same thing. The difficulties of the succeeding diagrams are graduated in such a manner, that each is almost as easy to do as that which preceded it, and the address acquired by practice is of advantage to the previous exercises. It is only necessary sometimes to recur to these, that they may not be forgotten.

N. B. To understand perfectly the particular rules we are going to lay down, the master should have the series of orders inscribed on the monitor's tablets before him, as also the plates affixed to this work; and he should proceed from question to question, and follow the successive explanations in the text, with constant reference to the diagrams.*

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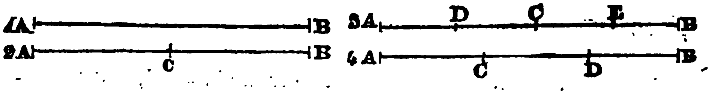
* This introductory part has been much abridged, but the remainder is a faithful translation from the original French work.

PARTICULAR RULES

FOR

THE USE OF THE MASTER.

FIRST CLASS—FIRST PART.



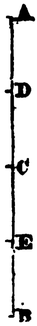
The first class draws only right lines, triangles, and perpendiculars; it is divided into two parts. The corrections are made with the rule, and occasionally with the square.

The first four figures, which are marked above, refer to questions No. 1 to 14, on the monitor's tablet; it is sufficient to see to understand them, and they require no explanations. At first it will be better that the monitor should draw the line with his ruler, and that the boys should divide it in equal parts; afterwards, they should trace the line themselves, (without a ruler) and divide it.

Nos. 9, 10, 11. These questions differ in nothing but change of terms from those of Nos. 2, 3, and 7. The children may also be exercised to find the three-fifths, three-eighths, &c., that is to say, to cut the line in five, or eight parts, and mark off three of them.

12, 13, and 14. These three problems relate to the use of the metre, and its subdivisions. We have already remarked on the accuracy which the eye can acquire, by familiarizing the child to trace these measures.

15. *Draw a horizontal line, &c.*—Hitherto the children have been allowed to trace their right lines in the direction that afforded them most facility; here the straight line on the black tablet is required to be drawn *horizontally.* The monitor verifies it by applying his rule perpendicularly to each end, and seeing that the ends are at equal distances from the bottom-edge of the tablet; and when he comes himself to draw the horizontal line which serves for a general correction, he makes the two ends equally distant from the bottom, and applying his rule to the tablet, draws a straight line through both points.



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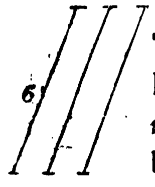
16. *Draw a vertical line, fig. 5.*—The *vertical* is a right line in the direction of a plomb line; the two extremities should be equally distant from the lateral edge of the tablet, which the monitor verifies in the same manner as the former. The monitor should repeat the thirteen first questions, applying them first to the horizontal, and then to the vertical lines; and the same observation should be made on the eighteenth question relative to oblique lines.

It is difficult to divide a vertical line in equal parts, as an optical illusion tends to make the upper divisions appear shorter than they are.

17. *Trace equi-distant horizontal lines, fig.* 1 *to* 4.—By measuring with the rule, the divergence of the lines at their extremities, it will be seen whether they are equi-distant in their whole length.



18. *Draw an oblique line, fig.* 6.—The monitor must not permit these lines to be in the same direction as the slant of common writing; which would be too easy. To divide these oblique lines in equal parts, remark the observation made on question 16.

19. A line is said to be *parallel* to another, when the right lines remain at equal distance from each other, in their whole length; and which is ascertained by applying the rule to each extremity of the lines, and finding that the divergence of one line from the other, is equal at both ends.

To draw exact parallels, the monitor should therefore trace the first line, and applying his rule to each end, mark two points at equal distances; then draw a line with his rule through each of these points, which will be the required parallel. See Problem 23.

20. *Join two points A B by a straight line, fig.* 1.—The monitor must mark two points in any direction, and the scholar trace a straight line from one to the other.

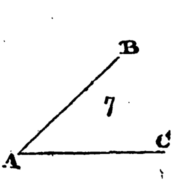
21, 22. Requires no explanation; it is necessary to exercise the children to draw straight lines in every direction.

23. *Trace a line parallel to another, through a*

given point.—After having described a right line, the monitor must mark a point, through which the scholar shall draw a second line parallel to the first. The position of this point must be considerably varied.

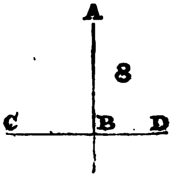
To draw parallels by the square, the monitor must place it on the tablet, so that one side of the triangle shall touch the line to which a parallel is required. Applying his rule to the other side of the triangle, he must lean on it with his left hand to keep it steady, and slide the square the length of the line, so that the rule shall always rest on it. The rule will then be in a position parallel to the line, and a line traced by it will of course be parallel to the first. As the monitor will find it difficult at first to keep the rule steady, he will do well to get one of the boys to assist him, by leaning against it, while he slides the square.

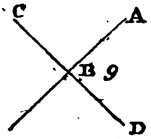
This process being generally used, it is necessary it should be shewn. The monitor only should draw with instruments; but the children, in seeing them applied, learn their use, and thus become able to employ them, and also to act as monitors in their turn.



24. *Make an acute angle, fig.* 7.—It is necessary to distinguish an angle from what is named a point or vertex. The angle is the opening or divergence of a line, A B, meeting another, A C; the vertex is the point A, where these lines cross each other. A compass, in which you gradually

draw apart the two legs, forms in this manner a multiplicity of different angles. It is the degree of divergence of the sides, which constitutes the size of the angle, and not the length of the lines A B, A C, which you may suppose to be prolonged infinitively.



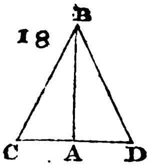


Conceive two lines crossing each other, as in figures 8 and 9, these will make four angles. These angles are right angles, if they are all equal, that is to say, if A B does not lean more on one side than on the other, so that in folding the figure by the line A B, the straight line B C will lie upon the straight line B D; it is then said that A B is perpendicular upon C D. But if A B leans towards B C, the angle on one side is then less than on the other, and in folding the figure by the line A B, the side B C, no longer falling on that of B D, the space on one side will exceed that on the other. The smallest angle is called acute, see *fig*. 7, the other called obtuse. Therefore, a right angle, *fig*. 9, is formed by two perpendiculars. An acute angle is smaller, and an obtuse angle larger, than a right angle.

25. ***Make an acute angle, whose opening, &c.***—It is required here to execute the different positions of the angle, *fig*. 7.

26. *Form a triangle.*—The space included between the sides of an angle remains indefinite at the opening; close this space by a right line, and you will form a triangle, B C D, *fig.* 18, a figure which has three sides and three angles; the *base* C D, being considered one of its sides on which it is supposed to rest; the *vertex* B, of the triangle, is that of the angle which is opposite to the base; the line A B, is the perpendicular drawn from the vertex to the base. A triangle is said to be an *isosceles* triangle, when it has, like that of *fig.* 18, two equal sides, C B equal to D B; if the three sides are equal, it is an *equilateral* triangle; if all the sides are unequal, it is a *scalene* triangle.

18

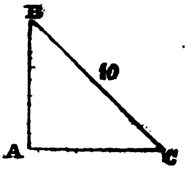


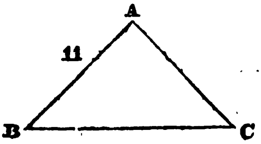
FIRST CLASS—SECOND PART.

1. ***Draw a horizontal and a vertical line, &c.*** *fig.* 8.—The children having already learned to draw horizontal and vertical lines, they will execute this right angle with great facility. The four problems which follow, are for the purpose of teaching to draw right angles in every position; the monitor proves the accuracy of the lines, by

applying the square, or by the problem, No. 11, by means of his rule only.

2 and 3. *Draw a perpendicular, &c., trace two lines at right angles.*—These two questions only differ from each other in their terms, The child traces a line, C D, *fig.* 9, in a direction which the monitor indicates with his rule, afterwards crosses the right line by one perpendicular to it.

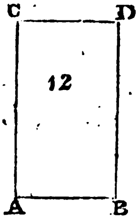


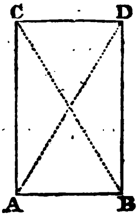


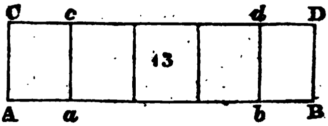
4. *Draw a rectangle triangle, fig.* 10 *and* 11.—A triangle is thus named which has a right angle A, that is to say, the two sides A B, A C, perpendicular one on the other. The base may be either horizontal or inclined, or you may take for your base, either one of the sides of the right angle (A C, *fig.* 10), or the longest side (B C, *fig.* 11). The pupils must be exercised to make these figures in every direction.

5. *Draw a rectangled isosceles triangle, fig.* 10.—It is necessary that the sides of the right angle of the rectangle triangle A B C, should be equal in *fig.* 10; the lines A B and A C are equal.

fig. 12 *bis.*





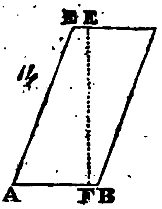


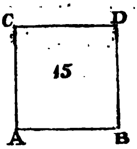
6. *Draw a rectangle, fig.* 12.—We name thus, the *fig.* A B C D, which has four sides, of which the opposite sides are parallel and equal to each other, and its four angles right angles; it is what workmen call a long square. A B is named the base, A C, the height; care must be taken to assign specified lengths to these two lines, thus for instance, to desire a rectangle to be traced of 12 centimetres in the base, and 21 in the height.

A remarkable property in this figure, which is much used in practice, and by which it may be ascertained if the angles are at right angles, is, that the lines A D C B, (*fig.* 12 *bis*), which cross from one angle to the other, are equal: therefore, the monitor will easily prove it with his square, or even with his ruler, since A B should be equal to C D, A C to B D, lastly, A D to B C.

7. *Draw a rectangle, and divide it in equal rectangles, fig.* 13.—It is only necessary to draw perpendiculars at equal distances from each other, such as *a c, b d,* &c.

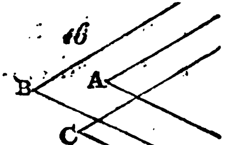
8. *Draw a parallelogram, see fig.* 14.—The parallelogram has, like the rectangle, its opposite sides parallel to each other, but it may not have any of its angles at right angles. The perpendicular E F is the *altitude,* A B the *base.* The rectangle, the square, and the lozenge, are species of parallelograms. See Nos. 9 and 18.

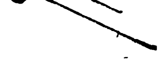




9. *Draw a square, fig.* 15.—This figure has its four sides equal, and its angles at right angles.

Therefore, to recapitulate, if the opposite sides are simply parallel, *fig.* 14, with or without right angles, the figure is a parallelogram; it is a rectangle (*fig.* 12,) if the sides are perpendicular; and a square (*fig.* 15,) if they are besides all four equal. Lastly, when the four sides are equal, but that no angle is at right angles, the figure is a rhomboid, or lozenge, (see Problem 18). As these different figures consist simply of parallel and perpendicular lines, they are readily verified, and the figure, which the monitor should himself draw in his turn, is easy to comprehend.



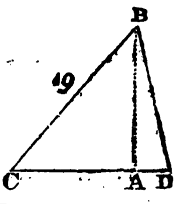


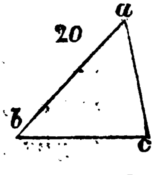
10. *Draw two angles with parallel sides, fig.* 16 *and* 17.—After having drawn an angle, a second is traced by drawing two lines parallel to the sides. The length of the sides is arbitrary, the size of the angle not depending on it. Two angles, like those in *fig.* 17, are said to be equal, not when their sides are of equal length, but

when the opening of one being applied to the other, they will fit exactly in the breadth of the opening, though one may be longer than the other from its vertex. *Fig.* 16 is intended to exercise the children in drawing angles with parallel sides in all sorts of positions.

11. *Draw oblique lines at equal distances from the perpendicular, (see fig.* 18, *page* 23).—In order that the two obliques, B C, B D, should fulfil the prescribed condition, they must be of the same length, or the distances, A C, A D, at the base of the perpendicular should be equal. Nothing is more easy to conceive or to draw.

12. *Draw an isosceles triangle, (fig.* 18, *page* 23). The triangle B C D, which has two sides, B C, B D, equal to each other, is an isosceles triangle. The perpendicular, A B, drawn from the vertex on the base, should divide this base in half; the angles, C and D, should have the same opening turned contrary ways. Therefore the pupil should draw the straight line C D, on the middle of which he will trace the perpendicular line A B; after which he has only to draw the lines B C, and B D.

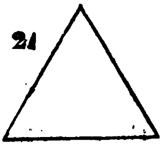




13. *Draw a scalene triangle, a b c, fig.* 20, *afterwards another, A B C, fig.* 19, *the sides of which*

shall be parallel to those of the first.—This is easy to conceive, and requires no explanation. The opening of the angles are equal in both triangles, and these triangles are called *alike.* The monitor should require that the first triangle a b c, should have its sides of a specified length, and that the second triangle should have its base of a given length, After having dictated the question, the monitor should name the number of centimetres, of which the base should consist, &c. &c.

21

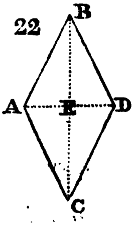


14. *Draw an equilateral triangle, fig.* 21.—The three sides are of equal length; and the monitor should name the number of centimetres which they should each of them measure.

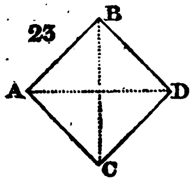
15, 16.—*Draw a perpendicular line through a given point, fig.* 18.

17. *Draw a perpendicular line A B, at the end of the line A C, fig.* 10.—These three questions differ from the second and third, only that in these the perpendicular is required to pass through a given point, which makes the execution of it more difficult. The pupil must draw his right line, mark the given point, and then trace his perpendicular line.

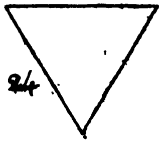
22



18. *Draw a rhomboid, or lozenge, fig.* 22.—The four sides are equal, as in the square, but the angles are not at right angles; draw two perpendiculars crossing each other, A D, B C.

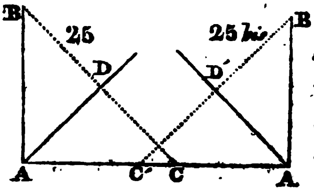


Let B E be equal to C E, A E be equal to D E, the lines drawn to connect the four points A C B D, will determine the figure of the lozenge A B C D.—If the perpendicular lines A B, C D, are both of equal length, and at equal distance from the centre, the figure will be a *square placed obliquely*, which is the question 19, fig. 23.



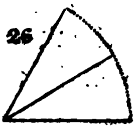
20. *Draw an equilateral triangle, &c. fig.* 24.—See what has been said, problem 14.—The equilateral triangle may even be placed in different positions, which gives rise to problem 21.

21. The monitor names the direction to be given to the base line, and the number of centimetres which it should contain.



Divide a right angle in half, fig. 25.—In order that the angle A should be divided in two equal parts by a right line A D; the line A D, must be as much inclined towards the side A C, as the side A B; if one conceives the figure folded on the line A D, each part will coincide, A B will lie on A C.

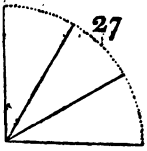
Let the monitor take two equal parts, A C, A B, let him draw the line B C; the middle of B C will be the point D, where A D divides B C: B C will be perpendicular to A D, and the triangle A B C will be an isosceles triangle, as in the problem 12, *fig*. 18, (page 23); he will therefore easily verify it, if A D divides the given angle equally, by measuring B D and D C, which he should find equal to each other. The *fig*. 25, *bis*, is the same as 25, but turned contrary ways.

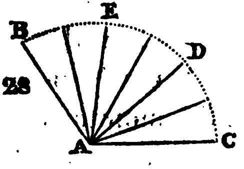


23. If the given angle should not be at right angles, *fig*. 26, it will be more difficult to divide in half; but the verification will be performed in the same manner.

24. *Double an angle, fig*. 26.—The pupil must draw a first angle by two lines A C, A D, *fig*. 25, afterwards he should draw A B, and the line A D must divide the angle B A C in half, which may be verified as in the former, and when the monitor wants to trace the same figure exactly, he must draw A D A C, *fig*. 25, forming the angle he is to double; then tracing on A D, the perpendicular line B C, he will take B D equal to D C; and through the point B thus determined, he will draw A B.

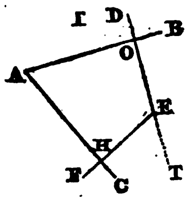
25, 26. *Triple an angle, fig*. 27; *divide an angle in three or in six equal parts, fig*. 28.



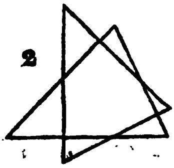


Nothing is more easy to conceive than these questions, but they are not so easy to execute. They are given to the children only as exercises, which contribute to form their accuracy of eye; but the monitor not having yet the use of the protractor, will not be able to verify them, nor to trace the figure himself, with the certainty of being correct.

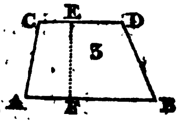
Second Class.



1. *Draw two angles with perpendicular sides, fig.* 1. — After having traced an angle B A C, the pupil must draw the line D E perpendicular to the side A B, then E F perpendicular to the side A C; the angle D E F, thus formed, is the angle required. It is to be remarked, that both the opposite angles A E, are acute, the others obtuse; and that if you prolong one of the sides, such as D E to T, the opening, F E T, is the same as that of the angle A.

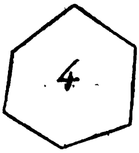


2. *Draw two triangles with perpendicular sides, fig.* 2.—After having formed a first triangle, a second is to be produced by tracing, first, an angle with perpendicular sides as above, and then closing it by a perpendicular line on the third side.



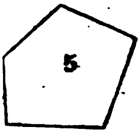
3. *Draw a trapezium, &c. fig.* 3. The figure A B C D, is named a trapezium. It has four sides, of which two are parallel, and are

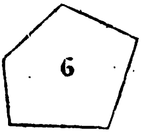
called the *bases*, as C D and A B; the *altitude* E F, is a perpendicular from one to the other. The child must first trace the altitude E F, then the perpendicular lines C D, A B; each line being of a specified length in centimetres; lastly, he must close the figure by the lines A C, B D.

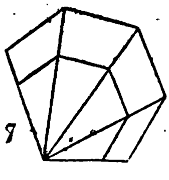


4. *Draw a polygon of 5 or 6 sides, &c. fig.* 4 *and* 5.—The pupil must first mark the vertices of these polygons, then join these several points by right lines, producing a closed figure; or else he must trace successive sides, and to each the monitor should assign a specified length, and even the direction of the sides, by applying his ruler at a distance from the tablet. When only one side remains to be traced, the pupil must close the figure.

5. *Draw two polygons with parallel sides, equal or unequal, fig.* 5 *and* 6.—This requires no explanation.

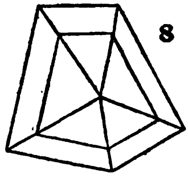






6. *Draw a polygon and its diagonals, &c. fig.* 7.—After having traced a polygon, the child must conduct straight lines from one of its angles to each of the others, traversing its surface, and dividing it into triangles; these right lines are what are named *diago-*

nals. There will then only be to trace parallels to the different sides, such as are seen in *fig.* 7, and which are limited by the diagonals, and there will thus be formed a polygon similar to the first. The monitor may require that the interior polygon shall be first traced, and the exterior one afterwards; in which case the diagonals must be prolonged beyond the first polygon.



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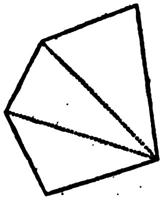
Instead of the diagonals being taken from one of the angles, they may be drawn from a given point in the interior of the figure, as is seen in *fig.* 8. The 7th question will then be resolved, viz. *draw a polygon, and from an interior point conduct lines to each of the angles.*

9



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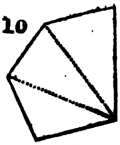
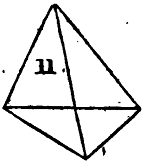
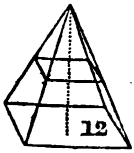


8. *Trace a polygon and diagonals, fig.* 9, *make as many triangles, fig.* 10, *with parallel sides.* The 9th figure is easily executed; to trace afterwards the 10th figure, the child must first draw one triangle with parallel sides to one of those in *fig.* 9, he must afterwards add a second, and then a third triangle to his figure; still keeping his sides parallel to those of *fig.* 9, he will thus have constructed a second polygon similar to the first.

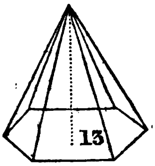
9. *Shew the height and breadth of the hectolitre of grain.*—The two dimensions are equal, they are 50 centimetres, and one third.

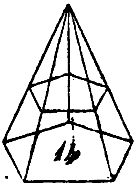




10. *Construct a triangular pyramid, fig.* 11. *Quadrangular pyramid, fig.* 12.—When right lines are drawn from a point, or common vertex, and terminate at the angles of a polygonal base, the figure thus formed is a pyramid: all the following figures, in the tablet of the second Class, are perspective; the figures, 11, 12, 13, and 14, represent pyramids, whose bases are polygons of 3, 4, 5, and 6 sides. The *altitude* of the pyramid is a perpendicular line, which falls from the vertex on its base, as is seen in *fig.* 12, the base of the pyramid rests on the horizon; its altitude is a vertical line, which by means of perspective, may be placed on any part of the base. When the base of the pyramid is a regular polygon, and that the perpendicular line falls in the centre, the figure is a *right* and *regular* pyramid; such are the figures 12, 13, and 14.

The child must therefore draw a figure with 3, 4, 5, or 6 sides for a base, afterwards marking a point beyond this figure for a vertex, he must conduct a line from it to each of the angles; or he may trace as many lines from a common point taken as a vertex, and closing the space between them, form a polygon.



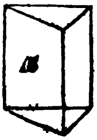


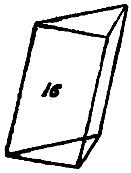
11. *Draw a right and regular pyramid, fig.* 13, and 14.—After having drawn a polygon of equal opposite sides, corresponding with each other, as in the *fig.* 13 and 14, draw a vertical line from the centre; take one of its points as a vertex: the pyramid has its angles placed to correspond at each side of the perpendicular line.

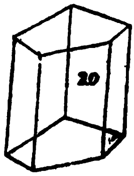
12. Divide a pyramid by a plane parallel to the base, *fig.* 12 and 14: the polygon thus formed, should have its sides parallel to those of the base. Trace lines from each angle of the polygon to their common vertex, on any point of one of these lines draw parallel lines to the base; the first lines forming the limit to which these parallel lines to the base should be drawn.

13. *Trace the frustum of a pyramid with parallel bases, fig.* 12 and 14.—Two polygons, similar and parallel to each other, must be traced, one above the other; the vertices must then be joined by right lines, and the frustum will be completed. These last lines being prolonged, must meet in a point. In fact this figure is the same as the last, if the parts of the lines which

join the vertex, and are above the upper polygon, be effaced.

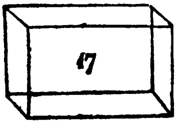


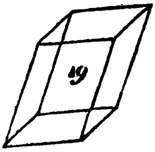


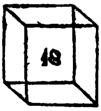


14, and 15. *Construct an oblique triangular prism, fig.* 16, *or right ditto, fig.* 15.—The prism is a body formed of two equal and parallel polygons; the similar vertices are joined by lines, parallel and equal to each other; such are the figures 15, 16, &c. to the 22nd. The altitude of the prism is a vertical line which terminates in two parallel bases. A prism is said to be a right prism, when the sides are perpendicular on the bases, *fig.* 15, 17, and 21.

The Child must draw a triangular base, then to finish the prism, he must trace parallel sides, and afterwards limit these lines by a second triangle, equal and parallel to the first. He will take other polygons for a base, and a similar construction will give the figures 20, 21. In general, if you draw right lines parallel and equal to each other, and that you join the upper extremities, so as to form a polygon, and also the lower extremities, the figure will be a prism, and the bases will be parallel and equal. The children should be well exercised to draw right lines, which diverging from one point, should terminate with precision on other given points; or else, fall perpendicularly in given directions.





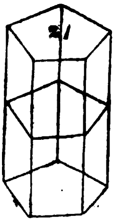


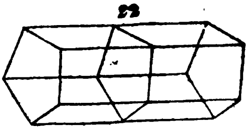
16, 17, 18. *Draw a right parallelipiped, fig.* 17, *or oblique ditto, fig.* 19.—When the base of a prism is a parallelogram, the figure takes the name of parallelipipede, the six sides of which are parallelograms, the opposite pairs being equal to each other.

19. *Construct a cube, fig.* 18.—The cube is a parallelipipede, of which all the sides are equal squares, each square being at right angles to the one contiguous to it; the twelve sides are equal and perpendicular, or parallel to each other. On account of the perspective, the diagram only shews the front, and back squares, those of the sides cannot be exhibited. The pupils should draw these two squares parallel to each other; the rest will be easily executed. The children should practise to draw cubes in every position, in the same manner as they drew squares in the first Class; and they should describe cubes without any angles, whether horizontal or vertical, when the monitor dictates it.

20. *Construct a cube placed obliquely.*

21, 22, 23. *Divide a prism by a plane parallel* *to its base, divide it in 2, 3, or 4 equal parts, double triple a prism, fig.* 21 and 22.

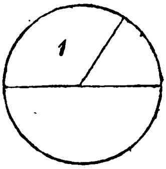


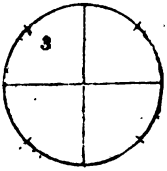


When a prism is divided parallel to its base, the section that is formed is a polygon, equal and parallel to the base. One must repeat here what was said on the sections made in a pyramid, *Nos.* 10, 11, and 12. When the plane divides the altitude in two equal parts, the prism is divided in half, and in the same manner in a third; and also to double the prism, it is only requisite to prolong the altitudes to another equal length.

In *fig.* 20, the prism is lying on the horizon, but the principle remains the same.

Third Class.





1. *Draw a circle, mark within it the centre, a radius, and a diameter, fig.* 1.—By regular practice, the pupils should attain to drawing circles,

and marking the centres of them with an accuracy equal to that of the compass. It is with this instrument that they should be verified by the monitor, and he should now be provided with it.

A *radius* is a right line which is drawn from the centre to the circumference; all the *radii* of a circle are equal. The diameter passes also through the centre, but it is terminated at both ends by the circumference, and divides the circle in two equal parts: a portion of the circumference is called an arc, or segment; and the right line which passes from one extremity to the other is a chord, or subtense.

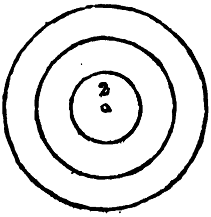
2. *Draw a circle, the centre, or the radius being given, fig.* 1.—Here the pupil should first trace either the centre, or the radius, in the position pointed out. It is then more difficult to execute the circle than when this condition is not required.

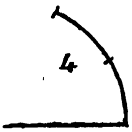
3. *Divide a circle by two perpendicular diameters, fig.* 3.

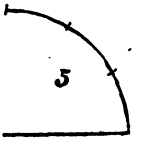
4. *Divide a circle into eight equal parts, fig.* 3. After having formed the circle, describe two diameters, one horizontal and the other vertical, and the circumference will then be divided into four parts. To divide it into eight, each segment should be again divided into half; which is easily done by dividing each right angle into two equal parts, as the children have already learned to do in the first part of the first Class, No. 22.

After a little practice, the monitor must require that the diameters should be in a given direction, so that neither of the two perpendiculars should be either horizontal, or vertical.

In joining the points thus obtained, by right lines or chords, you form squares or octagons, inclosed within the circle, which are the problems, 14, 22.







5. *Describe concentric circles.*—In *fig.* 2, all the circles have the same centre. It may besides be required that they shall all be equidistant; that is, that the diameter of the largest shall be cut in equal parts.

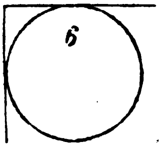
6. *Trace two circles, the diameter of one being double or triple that of the other.*—This requires no figure or explanation.

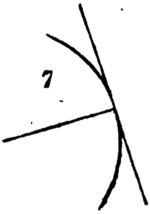
The monitor should always dictate how many centimetres the length of the radii of the circles should measure.

7. *Trace an arc of a circle, and mark its centre.*

8. *Trace an arc of a given radius, fig.* 4, and 5.—It is easier to trace a whole circle than a segment of one, as the eye judges better of the distance of the centre when the circumference is completely formed; except for this difficulty, these problems are the same as the preceding. The monitor should very much vary the radii and position of the centres.

9. *Divide an arc in half, or in three parts, fig.* 4 and 5.—This division of arcs into equal parts is of great use. The monitor should verify the figure by measuring it with his compass, if the distance of the points which are marked are the same, that is to say, if the chords of the arcs are equal.

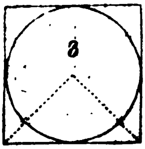


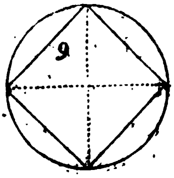


10. *Trace a circle, draw a tangent, fig.* 6.

11. *Trace an arc, draw a tangent, fig.* 7.—A tangent is a line which touches a circle, but does not cut it, or penetrate the interior of the circle; it only joins the circle in one point of the line, if drawn as fine as it should be. One peculiarity of the tangent is, that if a radius is drawn to the point of contact, the two right lines will be perpendicular to each other.

The monitor must verify this construction, either by applying his square, to be assured that the angle is a rectangle, or by marking two points on the tangent line, at equal distances from the point of contact, and finding, by his compass, that the points thus marked are at equal distances from the centre of the circle, as in the punctuated lines, *fig.* 8.





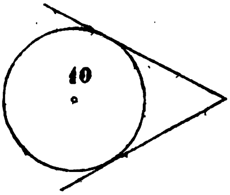
12. *Draw four tangents to a circle, forming a quadrilator, fig.* 8.

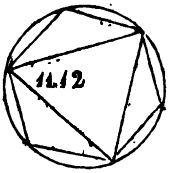
13. *Circumscribe a square on a circle, fig.* 8.—Four tangents surrounding a circle form a quadrilator, of which the monitor may give the length and position of two of the sides. When it happens, as in *fig.* 8, that the four tangents form right-angles to each other, the figure is a square, and right lines drawn to the centre should be equal, and at right angles. When a polygon is traced so as that all its sides shall be tangents to a circle, it is said to be *circumscribed* on the circle, or that the circle is *inscribed* within the polygon.

14. *Inscribe a square within a circle, fig.* 9.—When a square has its angles placed on a circumference, it is said to be inscribed within a circle, or that the circle is circumscribed on the polygon. There can be no difficulty here after what has been explained in problems 3 and 4.

15. *Double or triple a segment of a circle, fig.* 4 and 5.—An arc is first traced, and the centre marked; it must then be prolonged to the length required, still keeping within the line of the circumference, which is ascertained by the compass. This is more difficult than question 9. See observation after No. 37.

16. ***Draw a tangent to a circle from a given point, fig.*** 6 *and* 10.—In problems 10 and 11, the tangent is brought to a point marked on the circumference; but here the point is beyond the circle, and this line should be perpendicular to a right line drawn from the centre to the point of contact. It is to be observed, that two tangents to a circle may be drawn from a point beyond the circle, as in *fig*. 10, which is the object of problem 17. ***Draw two tangents to a circle from an external point.*** All these diagrams should be drawn with fine strokes, otherwise the tangent loses itself in the coarseness of the drawing, and the figure becomes confused. The position of the external point should be greatly varied.



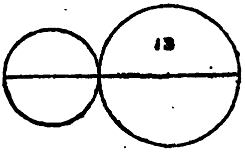


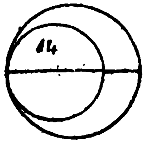
18. ***Divide a circle into six equal parts, form the regular hexagon inscribed, fig.*** 11.

19. ***Divide a circle into three equal parts, inscribe an equilateral triangle, fig.*** 12.—These two diagrams are formed into one figure, to shew the rule more clearly.

If you take the radius of a circle, and carry it from end to end on the circumference, you will find that the circumference just measures six radii: mark them, and join these points with chords, and you will form a regular polygon of six sides, which is called a hexagon; if you draw the chords only from every alternate point, you will have inscribed an equilateral triangle.

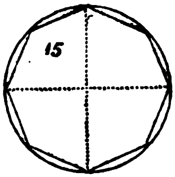
20. *Draw two unequal circles, the tangents external, fig.* 13, *the tangents internal, fig.* 14.

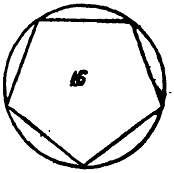




21. *The same figures, first giving the centres and point of contact.*—When two circles touch each other, whether internally or externally, the point of contact and the two centres are in a right line; this condition must be observed, and also the line of circumference be perfectly correct, to be assured that the two circles touch each other, the monitor always insisting on the pupil's making fine strokes. The manner of dictating this problem should be much varied, either by giving the radius, and leaving the pupil to choose the centre, or by marking first the centres, and the point of contact, or by giving a point through which the arc should pass, all these modifications by which this problem may be combined with the conditions of those which have preceded, and are to follow it, are left to the sagacity of the teacher.

Remark, that if you draw a tangent to the two circles at the point of contact, it will be perpendicular to the right line which joins the two centres.



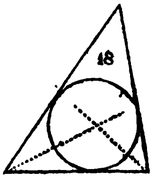


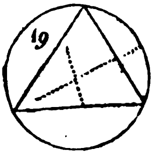
A regular polygon is that, which having all its sides equal, has also all its angles equal. When such a polygon is inscribed in a circle, the sides are chords of equal segments of the circle, and the vertices cut the circumference into equal parts.

22. *Inscribe a regular octagon in a circle, fig.* 15.—Draw two perpendicular diameters, divide each quarter of the circle into half, and join these points, see problem 3. Here the only difficulty consists in the necessity of drawing with fine strokes, which will not get confused at the vertices of the angles.

23. *Inscribe a regular pentagon in a circle, fig.* 16.—It is difficult to divide a circumference into five equal segments, and the object of this problem is to exercise the pupils in doing so, the accuracy of the division is ascertained by the compass.

24. *Trace a regular polygon of five, six, and eight sides, without forming a circle, fig.* 16, 12, 15.—There is great difficulty in executing this, but none in comprehending, or in verifying it.





25. *Form a triangle, and describe the circumscribed circle, fig.* 19.—First trace a triangle, after which describe a circumference, which shall pass through the three vertices. This figure is difficult to execute; to assist in doing so, draw a perpendicular line on the middle of one of the sides of the triangle, and a second from another side; these two perpendiculars will cut each other, and the point of intersection will be the centre: it is by this process that the monitor should draw his own figure, using the square and compass to execute it with perfect accuracy.

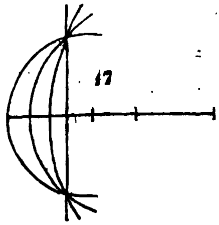
26. *Form a circle, and trace a tangent triangle, fig.* 18.—Three tangents to a circle, which shall inclose it in the form of triangle, are easy to trace; but the monitor must render it more difficult, by giving the required directions of the tangents; however, the tangents must always be perpendicular to the radius or right line drawn from the centre to the point of contact, as in the problems 11 and 16.

27. *A regular polygon being given, trace a circle which shall pass through all its vertices, fig.* 9, 11, 12, 15, and 16.—The difficulty of tracing a regular polygon without forming a circle, has already been conquered in problem 24; there only remains now to trace the circumscribed circle.

28. *Inscribe a circle in a triangle, fig.* 18.—This problem is the inverse of No. 26; the triangle is given, the tangent circle is afterwards to be

described, which presents a much greater difficulty. Remark, that if from one of the vertices you draw a right line, which divides the angle in half, and also draw a second line in the same manner from another of the vertices, the point at which these right lines intersect each other will be the centre of the circle. The monitor must form his own diagram by means of these lines: the radius of the circle is the perpendicular, which is afterwards drawn from the centre to either of the sides, for the three perpendiculars should be equal, see Obs. No. 37.

29. *Form an arc which shall pass through two given points, mark its centre and radius, fig.* 4 *and* 17.—After having marked two points, it remains to trace a segment of a circle which shall pass from one to the other. The centre should be found on some part of the perpendicular, in the middle of the chord which joins the two given points; but in fact, the centre may be taken on any part of that perpendicular line, so that there are an infinite multitude of arcs which pass through the same two points. The pupils must, therefore, be required to mark the centre of the arc, when they have acquired some facility in this problem. It is easy to conceive this multitude of arcs which pass through the two points given in *fig.* 17, and to understand the following problem, which should be executed with fine strokes of the pencil.



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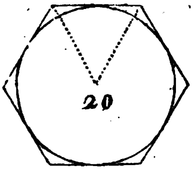
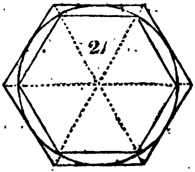
30. *Make several arcs, which shall pass through two given points, fig.* 17.—The centre of these arcs must be marked by the pupils, which centres are all on the perpendicular line which divides the chord.

31. *Form a circle which shall pass through three given points.*

32. *Form a circle to a given triangle, fig.* 19.—These two questions are the same as the 25th, but conceived in different terms. Among the multiplicity of arcs which pass through two of the given points, that one should be preferred, which has the property of passing also by the third, and there is but one which does so; the 25th problem shews the method of forming it, and which the monitor should adhere to, either to correct the drawings of his class, or to draw the figure himself. It is intended here, only to regulate the hand and eye of the child who, by practice, should arrive at selecting at a glance, among all these circles, that one, which passing through two given vertices, shall also pass through the third.—See Obs. No. 37.



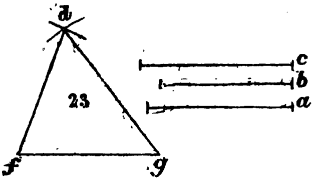


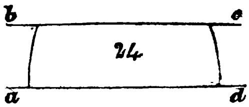
33. ***Having a circle given, to circumscribe on it a regular and irregular polygon, fig.*** **20.**—Draw different tangents to the circle which shall surround it, but if the polygon is to be regular, that is to say, if all the sides shall be equal, and the angles equal, the points of contact must be at the centre of the sides, dividing the circle into equal segments; in this case, the lines which lead from the centre to the angles, should divide them all in half, and be equal to one another.

To produce a regular circumscribed hexagon for example, the circumference is cut into six equal arcs, as if you were going to inscribe this polygon, and to each point of division is drawn a tangent. Radii may also be drawn to the six points of intersection, and be prolonged: then from each of the angles thus formed, lead a tangent to the centre of the intersected arc.

34. ***Having a regular polygon given, inscribe a cube, fig.*** **20.**—This problem is the inverse of the preceding one: trace first the regular polygon, afterwards describe the tangent circle to all its sides: the centre of the circle is found by dividing two angles into half, or else by drawing perpendicular lines from the middle of two sides, for all these right lines will meet in the centre.

35. *Inscribe and circumscribe to a circle, parallel polygons, fig.* 21.—After having traced a circle, inscribe a polygon; afterwards circumscribe another, by means of a succession of tangents parallel to the chords, and drawn to the middle of each arc; or the pupil may commence by the exterior polygon, and then proceed to the interior one. Observe, that if one of the polygons be regular, the other shall be so also; and that by prolonging the right lines which pass through the inscribed vertices from the centre, they will also pass through the circumscribed vertices.





36. *Form a triangle, the three sides of which are given, fig.* 23.—Trace three right lines at pleasure, *a*, *b*, *c*, which shall be the sides of the triangle (their lengths may also be given in centimetres); draw first the base *f*, *g*, equal to one of the sides as *c*. The vertex must then be found, so that its distance from *f* and *g*, shall be as the quantities *a* and *b*. To correct or construct this figure, the monitor must describe from the centres *f* and *g*, the arcs of circles *d*, with the legs of his compasses set by *b* and *c;* the point of intersection of these two arcs is the vertex sought for. It is to be remarked, that the two arcs of the circles may perhaps not cut each other; but in this case

the problem would be absurd, as no triangle could be formed. Each of the lines should be shorter than the length of the two remaining lines added together.

37. *From the point a, draw the right line a d, parallel to b c, fig.* 24.—From the centre *a*, describe the arc *c d;* then from the centre *c*, where this arc cuts the right line *b c*, and with the same opening of the compass, trace a second arc *a b:* lastly, mark the distance *a b*, on *c d*, and draw *a d*. This problem is of very general use, and when the monitor has executed it three or four times with the rule and compass, the children, whose sight has been formed to measure arcs, will be able to form this figure correctly, without the assistance of either of these instruments.

Observation.—It often happens that the same figure serves for different problems; for example, *to inscribe a circle in a triangle,* or *circumscribe a triangle to a circle,* we refer to *fig.* 18, and also *to circumscribe a circle on a pentagon, or inscribe a pentagon in a circle,* we equally refer to *fig.* 16. The geometrical methods of resolving these problems should be known to the monitors, and practised by them in their corrections; however, as it frequently happens in the problems that require the same figure, that there is one more easy than the others, the monitor may often be permitted to trace the figure by this last method, by which he will be enabled to correct the slates of his class with great rapidity; and the children having the exact figure before them, will attain the idea of accuracy that is required of them, which is all that is necessary.

FOURTH CLASS.

As the figures now become more complicated, in order to avoid a useless increase to the size and price of this volume, it has been considered advisable not to accompany the text with the diagrams to which it refers. It will therefore be necessary, in reading this and the following chapter, to study at the same time attentively, the sheets of engraved diagrams which belong to the fourth and fifth classes, and the teachers must learn to understand the figures according as they read the explanations.

1. *Trace a right line, which shall touch two circles, fig.* 1.—The radii drawn to the two points of contact, must be perpendicular to the tangent, and consequently parallel to each other. Observe, that the given circles may either intersect or be separated one from the other, the class should be required to execute the problem under both these conditions.

When it happens that the radii of the two circumferences, *fig.* 4, are equal, the tangent is then parallel to the right line which unites the centres; therefore, from two centres taken at pleasure on a given line, and with the same radius, trace two arcs; the right line, which shall touch both externally, will be parallel to the right line of the centres. It is also a method of resolving the following question, *draw a right line parallel to another right line.*

2. *Describe four tangents to two circles.*—It is sufficient to inspect *fig.* 1. to perceive how these four tangents may be drawn; two external, and two internal. It is to be observed, that the right line which joins the centres, contains also the point where these tangents intersect each other.

3. *Add two squares, fig.* 2 *and* 3.—These figures present two rectangle triangles, on the sides of which the three squares are constructed. The two small squares have this remarkable property, that one side of the square is precisely the side of the triangle, while the next side is a prolongation of the other side of the right angle; if to the longest side of the triangle you draw a semicircle, it should pass by the vertex of the right angle.

Now it is an axiom in Geometry, that the largest of these three squares contains a surface equal to that of the two other squares added together. Therefore, to add two given squares, form a right angle, and on its lines form the sides of these squares, these are the shortest sides of your rectangle triangle; draw the third side of the triangle to your right angle, and which will be the hypothenuse or longest side; this will be precisely the side of the square sought for. In finishing these three squares, as in *fig.* 2 and 3, you will have the addition you were required to make, *the large square equals the two small squares*, and the large square being given, the small ones are added, or the two small squares being given, the large one is added, which is equal to the two small squares. It is clear that if the two small squares are equal, the large square will be double of each of the small ones, as is seen in *fig.* 3; the problem 4. *Double a square* is therefore understood.

5. *Subtract one square from another.*—If on the contrary, the large square and one of the small ones are given, it is easy to find the other, in drawing the rectangle triangle according to these examples. The hypothenuse, or longest side, is first drawn, and this being taken for a diameter, trace a semicircle; mark a point on this semicircle, so that a chord drawn from it to the extremity of the diameter, shall be the length of the other given square; an opposite chord, which completes the triangle, will be the length of the square sought for. In forming the squares, *fig.* 4, the subtraction which has been made is exhibited, the great square, less by one of the small ones, gives the other as the remainder.

If you take a point on the circumference, at equal distances from the two extremities, the two chords will be equal, and the rectangle triangle is an isosceles triangle, as is seen *fig.* 3; so that the two small squares are here equal to half the large square; the sixth problem is therefore explained, *viz.*

6. *Take the half of a square, fig.* 3.

7. *Add three squares, fig.* 4.—The construction of *fig.* 2. serves to add two of the proposed squares, there remains to add a third to the sum; this requires another rectangle triangle, which shall have for one of the sides of its right angle the hypothenuse of the preceding angle, as is seen in *fig.* 4. One may afterwards add a fourth square, &c.

8. *Triple a square, fig.* 4.—This problem is easy to resolve, after what has been explained. When a square has been doubled, if to this double square you add the square proposed, you will have tripled this last; therefore, by repeating a second time *fig.* 2, you will have resolved the problem.

9. *Construct a protractor, fig.* 5.—It is agreed to divide all semicircles, large or small, into 180 equal parts, which are named degrees; a semicircle thus divided is named a *protractor*. After having traced a semicircle and its diameter, draw the perpendicular radius, and divide each quarter of the circle into 90 equal parts; it has already been shewn, that the right angle intersects at 90 degrees, the half right angle intersects at 45 degrees, from which it is said that a right angle is 90 degrees, a half right angle 45 degrees; in applying the radius on the circumference, it will contain just one-third of the half of the circumference, and each arc will be 60 degrees. The difference between this arc and the fourth of the circle, is 30 degrees, and with the half quarter 15

degrees; therefore, the half circle is in this manner easily divided into distances of 15 degrees; in cutting these arcs of 15 degrees in three parts, the curve will be divided into distances of five degrees: lastly, dividing these into five each, the protractor will be completed.

It is necessary for the proof, that the monitor should measure with a compass if the principal divisions cut the curve accurately into equal arcs. To save time, a protractor, whether in wood or brass, may be used for this purpose. The semicircle traced, may be either of a larger or smaller dimension than the instrument, but by applying this latter on the tablet, so that the diameter and centre of the protractor shall agree with these in the drawing, and passing a rule along the instrument through its divisions, one end of it being fixed in the centre of the protractor, you will ascertain the correctness of the figure.

10. *Produce an angle of* 36 *degrees, of* 50 *degrees, &c. fig.* 5.—Draw first a protractor, afterwards trace the radius which leads to 36° or 50°, &c.; the angle thus formed, will be that required. The obtuse angle, which is of the side opposite to the radius, will be 144° or 130°, by subtracting 36° or 50° from 180°. Observe, it is not necessary to draw the whole protractor, but to mark the gradations only which lead to those required; for 36° for example, the radius carried on the circumference gives an arc of 60°, its half is 30°, to which it is necessary to add six parts, or the fifth of the

arc of 30 degrees. The line traced, *fig*. 5, gives the angles of 36° and 144°. If it had been proposed to make the obtuse angle 144°, it is plain that it would have been sufficient to have made one of 36°, which would have given the other.

11. *Construct a sphere and its meridians*, *fig*. 6. —After having described a circle to represent the sphere, and two perpendicular diameters, of which one, *the Equator*, is a line at equal distances from the two extremities called *Poles;* and the other, *the axis* which connects the Poles; trace arcs of circles which shall all pass through the Poles, and the centres of which shall consequently be situated on the line perpendicular to the axis, prolonged to right and left, (3d Class, No. 30.) these centres of arcs are distant from each other, in proportion as they approach nearer to the axis. The number of these arcs is arbitrary, they must be formed like those in the diagram, more close to each other towards the exterior, and alike at both sides of the axis; these arcs represent different circles of the sphere in perspective, which are named *Meridians*.

12. *Construct a sphere, and the small circles which divide it into zones*, *fig*. 7.—After having described a circle, and its two perpendicular diameters, divide the circle into equal arcs, taken from the poles; for example, of arcs of from 15 to 15 degrees, join the points placed on the circle, at equal distances from the pole, by segments of circles, of which the concavity is turned towards

the pole, as is seen in *fig.* 7; the centres of these circles will all be on the axis prolonged beyond the sphere, and receding from the pole in proportion as they approach the equator, the space contained between the arcs is called a zone; these zones represent the small circles, which are obtained by dividing the sphere by a plane perpendicular to the axis.

13. *Draw an atlas.*—This figure is the same as the two preceding ones, united into one drawing, as if one had been traced on the other.

14. *Construct an Ellipse, fig.* 9 *and* 10.—An ellipse is the curved oval which is seen in the diagrams, and which may have a more lengthened form, such as *fig.* 11 to 15. First, draw two perpendicular right lines, take equal parts above and below, and the same to the right and left, from the point of intersection or *centre* of the ellipse; these parts, which in the circle are all equal to each other, are here only equal in pairs; there is, therefore, the great and small diameters of the ellipse, which are called its large and small axes: the extremities of the first are its two vertices.

It remains now to draw the curve, by imitating the diagrams, without any unevenness or interruption of the line; the four segments formed by the two axes, should be perfectly equal, inasmuch, that if you folded the figure on its axis, the parts when lying on each other, should exactly coincide; with respect to the rigorous outline of the curve, according to the methods of Geometricians, it is not necessary we should attend to it; it is sufficient if the graceful outline of the diagrams be faithfully imitated. The length of the two axes should be much varied, and the monitor should express them in centimetres.

15, 16. *Trace a cone, oblique, fig.* 11, *right, fig.* 12.—Take a circle for a *base*, and above this plane a point as a vertex: imagine a right line passing through this point, turning without quitting it round the circumference; this moving right line will describe a *cone.* It may be called a pyramid, whose base is a circle. A loaf of sugar is of a conical form. The perpendicular line from the vertex to the base, is the *altitude* of the cone; and if this perpendicular falls exactly in the centre of the circle, the cone is a *right* cone. The perspective in changing the apparent dimensions of the body, gives to the base of the cone the figure of an ellipse. The drawing can offer no other difficulty than that of forming the curve.

17, 18. *Draw a right cylinder, fig.* 13, *oblique, fig.* 14.—On two parallel planes, one above, and the other below, trace two equal circles, so that a right line or axis should pass through the centre of both. If you conceive a moving right line which, without ceasing to be parallel to the axis, shall turn on the two circumferences, the space inclosed by it is what is called a *cylinder*, it may be called a *prism*, whose bases are circles; the altitude is the distance which separates the two circular bases, that is to say, the perpendicular drawn from one to the other; if the axis should be this perpendicular, the cylinder will then be a *right* cylinder. The walls of a well form a cylinder. Here, as in the cone, the perspective changes the bases into two equal and parallel ellipses. The monitor must express the length of the axis of the curves, and the axis of the cylinder in centimetres.

19. *Draw a litre, a boisseau, an hectolitre.*—For grain and solid substances the height and diameter are always equal; for liquids the height is double the diameter.

The following are the heights or rather the depths.

	1. *For dry substances.*	2. *For liquids.*
Decilitre or poisson		8 centimetres.
Litre		17 cent.
Boisseau	25 centimet.	
Decalitre or velte	23 centimet. $\frac{1}{3}$	37 cent.
Demihectolitre	4 decimetres	63 cent. $\frac{1}{3}$
Hectolitre	50 centimet. $\frac{1}{3}$	80 cent.

20. *Draw a section parallel to the base of a cone or a cylinder, fig.* 11, 12, 13, *and* 14.—This section is a circle, which, seen in perspective, presents the appearance of an ellipsis; such as is drawn in the diagrams.

21. *Double a cylinder, &c. fig.* 13 *and* 14.—What has been before said on the prism, (2nd Class, No. 21, 22, 23,) must be repeated here, for the cylinder is only a species of prism with circular bases.

22. *Form a cylinder, whose axes shall be horizontal, fig.* 25.—The diagram sufficiently explains the meaning of this problem.

Fifth Class.

The figures of the fifth Class are composed by the combinations of those lines which have been formed in the preceding Classes, *viz.* the horizontal and vertical line, circular arcs, and elliptic arcs. It is a fact allowed by all persons of taste, that an outline can only possess grace and beauty in proportion as its parts form simple fractions of the whole; the half, the third, the fourth, are nearly the only fractional parts which please the eye; beyond these we can no longer estimate the proportions, and every thing becomes confused. We should adhere to those proportions and lines which are familiar and pleasing, for which reason curves should not be joined so as to form angles, and all sudden breaks in the line should be severely interdicted. Those superficies which are called *revolutions*, because they are conceived to be produced by the motion of a curved line round an axis, are the most pleasing to the eye; every section perpendicular to the axis, is found to produce a waving line, or that curve which has been styled the line of beauty. The outlines in the fifth sheet of Engravings have been drawn on these principles; all the lines necessary to assist in drawing the parts are dotted, the centres of the circles, the axes of the ellipses, are thus marked. The pupils must rub out their dotted lines when their drawings are finished; and as they become more expert, they should endeavour to form their outlines without this assistance of dotted lines.

As to the corrections, they should generally be made by the monitor, with the help of the ruler and compass; but as he must avoid wasting time on these operations, we recommend that one figure only should be drawn at each sitting of the class; which figure must be repeated again and again, (except the *fig*. 1, 2, 3, which are too simple to require this practice). The interest which the children take in the exercise, will prevent their feeling fatigue from these repetitions; by this means there will be but one correction to make at each sitting.

1, 2, 3. *Trace a filet, fig*. 1. *Astragal, fig*. 2. *Ovolo, fig*. 3.—These mouldings are so easy to conceive, that they require no explanation; they may be reduced to horizontal lines, vertical lines, and circles, of which the dotted line shews the radius.

4. *Draw a torus with its plinth, fig*. 4.—The torus, which is seen here in profile, is a large moulding usually placed at the base of columns. The torus has its diameter vertical and parallel to the axis of the pillar; it is produced by a semicircle which forms its revolution round this axis. The plinth is a short cylinder which supports the torus.

5, 6. *Draw a quarter round, or ovolo, with its filets, fig*. 5 *and* 6.

7, 8, *Draw a talon or ogee, straight or reversed, with its filets, fig*. 7 *and* 8.—The profile of the talon or ogee is formed by two arcs of circles, united end to end, and of which the centres are placed on the two sides of the right line which joins their extremities. This line which is dotted, is intersected in the middle by the arcs of the circles, and each half being taken as the base of an equilateral triangle, the vertex of this triangle forms the centre. The right line which joins the two centres, passes through the point of union of the two arcs, the middle of the first dotted line.

9, 10. *Trace a cyma recta, straight and reversed, fig.* 9 *and* 10.

11. All these mouldings are easily understood, they are generally executed in the position shewn in Plate V., but the pupils should be exercised to trace them in the opposite direction, facing them towards the left.

12. *Draw a flower vase, fig.* 11.—This vase is formed of parts, whose relations are perceived on mere inspection; right lines, and arcs of circles, are united in proportions which are easy to estimate.

13. *Construct a pedestal, fig.* 12.—Here the arcs uniting end to end, belong one to the ellipsis, and the other to the circle; the centres and axes are dotted.

The succeeding questions require no explanation, as an inspection of the figure will shew the different lines of which they are composed; one should conceive that all these lines turn round an axis to produce a revolving body.

PROBLEMS

OF

GEOMETRY AND ARITHMETIC.

The art of lineal drawing is of paramount utility to the lower ranks of society. The schools of mutual instruction are wholly devoted to elementary instruction, and since the science of calculation and acquaintance with geometrical figures have become parts of that instruction, it is necessary that these acquirements should not remain unprofitable to those who have obtained them. It is easy to unite two branches of education so important, and which have so much analogy to each other. Artisans should be enabled to make the calculations relative to their work, under whatever form it may have been executed; to draw out their own estimates, to calculate the price and the quantity of materials necessary; in a word, to make all the valuations which relate to the trade they follow. Under these considerations we shall unite in one system the simple principles of Geometry and calculation, and the series of rules and problems which are most frequent; to which we shall subjoin numerical examples, to shew the application of these principles. The teacher must copy these rules, which should afterwards be copied by those scholars who have passed through the five Classes of drawing, and are the most expert at calculation; they must be formed into a Class of Arithmetic. This occupation will not only be agreeable to the children, but will take off from the dryness of calculations, in which there appears no object, by occupying them with problems which are manifestly useful.

The rules to be copied are printed in italics in the most concise form, and to each a numerical example is given, which also shews its application. It is for the teacher to see these examples executed, and to multiply them greatly, by changing the figures at his pleasure, but still retaining the same form in which we shall state them. These little problems should be given by the master in writing.

It is to be regretted that the instruction of arithmetic in the schools should be so confined; had it been more extensive, there are a multitude of useful and curious questions which might have been included here; such as proportional lines, square roots, &c. But as this is not the place to reform this part of elementary instruction, we have confined ourselves to those problems which may be resolved by the four common rules. This has very much contracted the usefulness of the work, but such as it is, we hope it may fulfil the object we have had in view, and only mention what we consider a defect, with a hope to engage those who direct the instruction of arithmetic, to extend the bounds within which they have confined themselves.

Before we proceed further, let us shortly repeat the names of the measures, the manner in which their subdivisions are expressed in cyphers, and the process by which they are calculated.

There are five different measures: the unity of length is called *metre;* that of surface, *are;* the unity of wood measure, *stere;* that of capacity, *litre;* and lastly, that of weights, *gramme.*

To denominate the multiples of these unities, you place before them the words myria, *ten thousand;* kilo, *thousand;* hecto, *hundred;* deca, *ten.*

The subdivisions are designated by mili, *thousand;* centi, *hundred;* deci, *ten.*

These few words suffice to denominate all the measures; they are subdivided by decimal numbers: the myriogramme is worth ten kilogrammes, the kilogramme ten hectogrammes, &c.

The numbers are written in the same geometrical manner as decimal fractions; a stop being placed to the right of the number which expresses the whole numbers chosen as unity, which stop separates the fractional parts to the right. The place which the stop occupies, is of great importance. If I have to express 15 metres 34 centimetres, I may either write 15,34 metres, or 153,4 decimetres, or 1534 centimetres; in displacing thus the stop, you make the number 10 or 100 times greater; but the unity employed to express the number being 10 or 100 times less, the language and writing has changed without altering the value. In the same manner 543,27 litres are equal to 54,327 decalitres, and 5,4327 hectolitres. The teachers must study these numbers so as to be able to read them with facility; but the Classes will readily acquire them by the practice which their problems will afford.

In Addition and Subtraction, those parts which are of the same denomination, should be ranged under the same column; and the sum is generally conducted as if there were no decimal fractions: the stop is afterwards placed in the range, and under the column of stops. The following examples will explain this rule.

Addition.	*Subtractions.*	
432,1784	324,15	30,4
17,231	187,3	19,28
9,4	———	———
83,512	136,85	11,12
7,0		
———		
549,3214		

Multiplication is also conducted as if there was no stop, but it is placed in the product so as to separate as many decimals to the right, as there were in the multiplier and sum to be multiplied united; to multiply 4,37 by 2,3, I proceed as if I had the number 437 and 23, which gives the product 10051; I reckon two decimals to one of the numbers proposed, and one to the other, in all 3; I therefore separate three cyphers in the product, which now gives 10,051.

4,37	183,2
2,3	0,24
1311	7328
874	3664
10,051	43,968

For division, add zeros to the right of the number proposed which has the fewest decimals, so that the divisor and dividend shall have an equal number; suppress the stop, and divide in the usual manner; as soon as the quotient of the whole numbers is procured, place the stop; there now remains to find that of the decimals: put a zero to the right of the remainder, and divide it, you will obtain the first cypher after the stop; put another zero again to the remainder, and divide, you will obtain the second decimal fraction; in a word, you proceed as if you carried a zero from the dividend to each remainder. To divide 10,051 by 4,37, I write 4,370, and I divide 10051 by 4370.

4370) 10051 (2,3	21,2) 154,3 (7,27
8740	1484
4370) 13110	21,2) 590
13110	424
Remainder 0	1660
	1484
	176 &c.

When two or three decimal fractions have been obtained, you stop, as it would be useless to proceed further, the fractions being so small, as is seen in the second example. We will now proceed to shew the application of these principles.

1st. ON LINES.

PROBLEM.—*Find a side of a rectangle triangle, the two other sides being given.*

Multiply each of the known sides by themselves; add them together, if you are to seek the longest side, or hypothenuse; subtract, if one of the short sides; that done, you will have the same result as if you had multiplied by itself the unknown side; therefore you must seek a number, which, multiplied by itself, gives the product.

EXAMPLE.—The two sides of a rectangle triangle are, one 3 metres, the other 4; find the hypothenuse.

3 times 3 are 9	5 times 5 being 25,
4 times 4 are 16	the hypothenuse is 5 metres.
Added together . 25	

EXAMPLE.—The base of a rectangle is found to measure 8^{m}, 54; the altitude is unknown, but the hypothenuse has been measured, and found to contain 15^{m}, 32; what is the altitude? that is to say, find one of the short sides of the triangle.

8,54	15,32		234,7024
8,54	15,32	I subtract . .	72,9316
3416	3064		161,7708
4270	4596		
6832	7660		
72,9316	1532		
	234,7024		

There remains to find a number, which, multiplied by itself, gives 161,771, in keeping to millimetres. A few trials will give nearly 12,719, which may be proved by multiplying this number with itself; the altitude required is therefore 12^m, 719.

Example.—Find the altitude* of an isosceles triangle, of which the base is 52 centimetres, and the equal sides 87: the perpendicular drawn from the vertex, cuts the base in half; it forms a short side of the rectangle triangle.

Half of the base	26	Long side	87		7569
	26		87	I subtract	676
	156		609		6833
	52		696		
	676		7569		

A few trials will yield 83 centimetres for the altitude required; 83 times 83 will give 6889.

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* The altitude of an isosceles triangle, is the length of the perpendicular line, drawn from its vertex to the base.

Problem.—*Find the circumference of a circle produced in a right line.*

Multiply the diameter by three, and add a seventh part; the product will be the circumference.

Example.—The diameter of a circle is 4^{m}, 523.

	4523
Multiply by 3 . . .	3
	13569
Add a seventh of 4,523 .	646
	14,215

The circumference measures 14 metres and 215 millimetres in a right line.

Example.—The breadth of a circular pond is 5,5 metres, what is the circumference?

Diameter	5,5
	3
	16,5
Add one-seventh . .	0,786
	17,286

The circumference is 17^{m}, 286.

Problem.—*Find the radius, the circumference being given.*

Multiply the circumference by 0,159, *you will have the radius.*

Example.—To find the thickness of a column, its circumference has been measured by a line, and has been found to be 2^{m}, 542. The sum gives 0^{m}, 404, or 404 millimetres for the radius.

Six figures are struck off to the right, as there were three in each given number: the double is 808 millimetres, the diameter of the column.

$$\begin{array}{lr} & 2{,}542 \\ & 0{,}159 \\ \hline & 22878 \\ & 12710 \\ & {,}2542 \\ \hline \text{Radius .} & 0{,}404178 \end{array}$$

2nd. OF SUPERFICIES.

PROBLEM.—*Find the superficies of a parallelogram.*

The superficies of a parallelogram is obtained by measuring the base by the altitude; that of a square, by multiplying the side by itself.

The base and altitude should be expressed in measures of the same denomination; for example, in metres or decimetres: when a superficies is to be calculated, you seek how many times it contains a square, of which the side is a metre or a decimetre, &c. The multiplication made, shews how many squares the superficies contains, of which the side of the square and the line of the superficies being both expressed by the same unity, whether metre, decimetre, &c,

The square metre contains 100 square decimetres, each of these decimetres contain 100 square centimetres, which makes 10,000 square centimetres in a metre.

EXAMPLE.—A rectangle has 2^m, 24 base, and 4,31 altitude, what is its superficies?

Multiply the two numbers together, point off four numbers in the product, these being two in each given number. The superficies contains then 9 square metres, and the 65 hundreds of one of these squares (that is, 65 square decimetres), and again 44 hundreds of these, or 44 square centimetres; since the square metre is composed of 100 square decimetres, of which each is composed of 100 square centimetres.

2,24
4,31
224
672
896
9,6544

Example.—A canal is 154,6 metres long, by 75,3 metres broad; how many ares does its superficies contain?

The product gives 11641 square metres, and 38 hundred parts; but the are being a square of 10 metres, contains 100 square metres; therefore, the canal is 116 ares and 41 hundreds of ares, (leaving out the 38 ten thousands).

154,6
75,3
4638
7730
10822
11641,38

Example.—A park is of a rectangular form, of 2023 metres in length, by 1145 broad; what is the extent of it? The product of the multiplication of these two numbers is 2316335 square metres, or 28163 ares and 35 centiares, or 231 hectares 63 ares and 35 centiares.

Problem.—*The superficies of a right angled prism, without including its bases, is found by multiplying the altitude by the outline of the base.*—This is the consequence of all the lateral faces of the body being rectangles, by which it is included in the preceding rule.

Example.—It is required to dash the outside walls of a park; these walls are 2,3 metres in height, how many superficial metres do they contain? The park forms a parallelipiped of 2023 metres long, by 1145 broad, and 2,3 metres high; I double the two first numbers, and add to find the length produced of the wall, I find 6336 metres of circumference; multiply this by 2,3, I get 14572,8 square metres.

Example.—It is required to cover the walls of a room with paper. The dimensions of the room are 24^{m}, 7; the height of the walls to be covered is 3^{m}, 1; the paper is 2^{m}, 5 broad: how many metres of paper will it require? I multiply 24^{m}, 7 by 3^{m}, 1, to find the superficies that is to be covered, and get 76,57 square metres. My paper must be of such a length, which multiplied by 1^{m}, 5, shall give the same result; I therefore divide 76,57 by 1^{m}, 5 (see page 68), I find 51,04, it accordingly takes rather more than 51 metres in length to paper the room required.

24,7	1,50) 76,57 (51,04
3,1	750
247	157
741	150
76,57	700
	600
	100

If the prism is oblique, its superficies is calculated by taking that of all the parallelograms of which it is formed.

PROBLEM.—*Find the superficies of a triangle.*

Multiply the base by the height, and take the half of the product.

A triangle is always the half of a parallelogram, which is of an equal base and equal altitude.

EXAMPLE.—It is required to find the extent of a triangular field, of which one side taken as base is 154 metres long, and of which the perpendicular, drawn from the vertex to the base, measures 83 metres. I multiply 77, the half of 154 by 83, and I have the superficies required, 6391 square metres, or 61 ares 91 centiares.

(Instead of halving the product, you may take the half of either the base or the altitude, before you multiply, which will produce the same result.)

The superficies of a polygon, that of a pyramid, is calculated by taking separately the superficies of each of the triangles of which it is composed.

EXAMBLE.—An irregular court has a quadraliter form; to find the superficies I measure one of the diagonals, which I find to be 129^m, 7: I draw perpendiculars on this line, from the angles opposite to it; I find one of them of 52^m, 5, the other of 41^m, 8. I consider the court as formed of two triangles, of which the superficies must be calculated separately.

First triangle .	129,7	Second triangle .	129,7
By	52,5	By	41,8

Two multiplications should be made; but I have operated quicker by adding the two altitudes 52,5 and 41,8, which gives 94,3. The half 47,15, should be multiplied by 129,7, which gives 6115,355 square metres, or 61 ares, 15 centiares.

If the polygon, whose superficies is to be found is regular, you draw lines from the centre to two of the adjoining angles, and calculating the superficies of the triangle thus formed, you repeat the product as many times as there are sides to the polygon.

Example.—An hexagonal pond has one of its sides which measures 3^{m}, 34; its breadth from the centre of one of the sides to the centre of the opposite side, is 2^{m}, 88; one of its triangles has then for its height 1^{m}, 44, and for its superficies the product of 0,72 by 3,34, or 2,4048. I repeat this 6 times, and obtain 14,4288 square metres, that is to say, 14 square metres, 42 square decimetres, and 88 square centimetres.

Problem.—*Find the superficies of a trapezium.*

Take the half of the sum of the two parallel sides, or what is equal, the sum of the parallel drawn at equal distances of these lines, and multiply by the height.

Example.—A roof has the figure of a trapezium of which the parallel sides are of 44^{m}, 7, and 33^{m}, 5; the height of this trapezium (measured on the roof in the direction of the slope, and perpendicularly to the two preceding sides,) is of 9^m, 4; what is its superficies?

	44,7	39,1
	33,5	9,4
Sum	78,2	1564
The half .	39,1	3519
		367,54

You obtain 367 square metres, and 54 square decimetres.

PROBLEM.—*Find the superficies of a circle.*

Multiply the radius by itself, and afterwards the product by 3 and ⅐ (*or* 3,143); *you will have the superficies of the circle.*

EXAMPLE.—A circular pond has a radius of 8^m, 3; this multiplied by itself, gives 69^m, 89; tripling this, and adding the seventh, you obtain 216 square metres, and 51 square decimetres, (nearly one square metre).

69,89
3
20667
984
216,51

EXAMPLE.—The circumference of a circular pond being measured, it has been found to contain 28^m, 5; it is calculated therefore (page 71) that the radius is 4^m, 53; multiplying this by itself, I find 20,52, which repeated 3 and ⅐ times, gives 64,49 square metres.

PROBLEM.—*Find the superficies of a right cylinder.*

Multiply the circumference of the base by the altitude.—As the base is a circle, the radius being given, the circumference is easily found, (page 71)

Example.—A painter painted the walls of a circular room, the walls were 3,4 metres in height, the diameter of the room 54,2 metres; how many square metres did he paint? Multiply 54,2 by $3\frac{1}{7}$, the product is 170,34 for the circumference of the room; I multiply this by the height 3^m, 4, and I find 579,156, that is, 579 square metres, 15 square decimetres, and 60 square centimetres.

We have not allowed for doors and windows in this calculation; they must be calculated by themselves, and subtracted. The mouldings and cornices are measured by long measure, a tape line marked in metres bends with their form, and the length thus measured gives the real dimensions of the work that has been executed.

3rd, Calculation of Contents.

Problem.—*Find the contents of a prism of a cylinder.*

Multiply the superficial contents of the base by the altitude, the product will be the cubic contents contained in the given body.

Note.—The three dimensions must always be expressed in the same denomination of measure, whether metre or decimetre, in the same manner that, in the calculation of superficies, it was necessary to use the same unity to express both the length and breadth.

Example.—A wall is 2^m, 8 high, 6 decimetres thick, and 104^m, 5 long; how many cubic metres of stone does it contain? I multiply these three numbers in writing 0^m, 6, instead of 6 decimetres, and I find

2^m, 8, multiplied by 0^m, 6, gives 1,68 square metres.
1,68, multiplied by 104^m, 5, gives 175,560 cubic metres.

I have therefore 175 cubic metres, and 560 cubic decimetres; (the cubic metre contains 1000 decimetres) of stones, not allowing for earth, sand, or mortar, which may have been used in the construction of the wall.

Example.—A pile of wood ranged in the form of a parallelipipede, is 22^m, 3 broad, 54^m, 8 high, and 37^m, 1 long; how many steres or cubic metres does it contain? I multiply the three numbers, and I find 45337 cubic metres, and 684 cubic decimetres.

Example.—A boiler, which is nearly cylindrical, is 8,3 decimetres deep, 13 decimetres broad; what is its contents? The radius is 6,5 decimetres; multiply 6,5 by 6,5, and afterwards by $3\frac{1}{7}$; I get for the superficies of the circle of the base 132,786 square decimetres. Multiplying this by the depth (or height), 8,3, I obtain the contents required, 1102,124 cubic decimetres, that is, (1102 litres) nearly 11 hectolitres.

Example.—Suppose a burnt brick to be 23 centimetres long, 11,2 broad, and 3,6 centimetres thick; the product of these three numbers is about 930 cubic centimetres. The cubic metre contains 1,000,000 centimetres; therefore, in dividing by

930, the cubic metre is found to contain about 1080 bricks. How many bricks will it take to build a wall 300 metres long, 2^{m}, 2 high, and 3,6 decimetres thick. Multiply these three numbers, (the last being written 0^{m}, 36,) you find that the cubic contents of the wall is 237,6 cubic metres; multiply by 1080, you obtain 256608, the required number of bricks. However, the mortar required to cement the bricks diminishes this quotient.

Example.—A well is 6^{m}, 9 deep, 1^{m}, 2 in diameter; it is required to give 4 decimetres in thickness to the wall. How much stone will it take to build the wall that surrounds it? I calculate the wall as if it was to be built solid, which gives a cylinder, with a radius of 6 decimetres, adding 4, that is to say, 1 metre afterwards: I subtract the hollow part which forms another cylinder with a radius of 6 decimetres.

1st. Cylinder; 1^{m} multiplied by 1^{m}, and by $3\frac{1}{7}$, gives 3,14 square metres for the superficies of the base (which must be multiplied by the height 6^{m}, 9) . . . . 3,14 sq. met.

2nd. Cylinder; 0^{m}, 6, multiplied by 0^{m}, 6, and by $3\frac{1}{7}$, gives for the circle of the base 1,13

Which subtracted gives . . 2,01 sq. met.

It remains to multiply this by 6^{m}, 9, (the multiplication may be performed either before or after the subtraction) and I obtain the product 13^{m}, 869, nearly 14 cubic metres.

Problem.—*Gauge a cask.*

Take the superficies of the circle of the base, and twice that of the circle of the cask at the bung-hole; add these two numbers, and multiply the sum by the third of the length of the cask.—All these measures should be taken within the barrel, otherwise the thickness of the wood would be included in its volume.

Example.—A cask is 61 centimetres broad at the bung-hole, and 56 at its base, its length is 93 centimetres, what is its capacity? The radiuses are 30,5 and 28.

28 times 28 multiplied by $3\frac{1}{7}$ . .	. 2464	sq. cent.
30,5 times 30,5 . ditto	. 2923,64	
	2923,64	
Sum, leaving out fractions . .	. 8311	sq. cent.

Multiplying this sum by 31, which is the third of the length, you obtain the product 257641 cubic centimetres. As 1000 of these centimetres form the litre, the capacity of the cask is 257 litres, and 641 cubic centimetres.

Problem.—*Find the contents of a pyramid or of a cone.*

Multiply the superficies of the base by the height, and take the third of the product.

Example.—A loaf of sugar is 2,4 decimetres broad at its base, 4,1 decimetres high, what are its contents? The superficies of the base is 1,2 multiplied by 1,2 and by $3\frac{1}{7}$, which gives 4,52 square decimetres; this multiplied by 4,1, (the height) gives 18,532; the third, or the volume, therefore, is 6 cubic decimetres, and 177 cubic centimetres.

PROBLEM.—*Find the contents of the frustum of a cone with parallel bases.*

1st, *Multiply by itself each of the radius's of the two bases, and then multiply them by each other;* 2dly, *Add then three products;* 3dly, *Multiply the sum by the height, and add to the product the 3rd of its 9th, (that is to say, its 27th.)*

EXAMPLE.—A tub is 2,9 decimetres broad at top, and 2,3 decimetres broad at bottom; the perpendicular depth is 3 decimetres, what is its volume?

1,45 multiplied by 1,45 . gives . .	2,1025	
1,15 multiplied by 1,15	1,3225	
1,45 multiplied by 1,15	1,6675	
Sum, leaving out the ten thousands .	5,093	sq. dec.
Multiplied by 3, the height	15,279	cub. dec.
The ninth of this number is 1,698, of which the third is	0,566	
	15,845	

Therefore the contents of this tub is 15 cubic decimetres, and 845 cubic centimetres.

And if it is required to know how many of these tubs full, would fill the boiler, which (page 79) has been found to contain 1102,124 cubic decimetres. This number must be divided by 15,845, the quotient, 69,55, shews that the boiler contains nearly 70 of these tubs full.

ON WEIGHTS AND MEASURES.*

WE have already explained (page 66) the system of their nomenclature, and their decimal subdivisions; it remains to explain how these may be obtained. It is necessary to be able to find the metre length, and there are many ways by which this may be accomplished.

The metre is *nearly* equal to three feet four inches and a half; to find the *exact* length of the metre, suspend a bullet by a thread to a nail fixed in the wall, move it gently out of its vertical position, and let it oscillate without touching the wall. If in counting the number of the oscillations, you reckon 60 in a minute, the *pendulum* will be precisely a metre in length, measuring from the point of suspension to the centre of the bullet. To be still more exact, there should be 361 oscillations in 5 minutes: when there are fewer oscillations, the thread should be lengthened, and if more, it should be shortened; till the proper number is obtained.

The French measures are so interlinked with each other, that the knowledge of one leads to all the others; therefore, having once acquired the ength of the metre, it has been shewn (page 60) how to find the length and breadth of the hectolitre, the boisseau, &c., and reciprocally, if you have one of these measures, it is easy to find the metre.

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* The standard of English weights and measures has been substituted here in place of the French weights and measures, and methods omitted which could not be practised in England, referring to French coin.

It would be a very useful exercise to calculate the volumes of the cylinders which form the measures of capacity; in taking the data given (in page 60), it will be found that

The hectolitre is equal to	100	litres of cubic decimetres.
demi-hectolitre . .	50	
decalitre	10	
boisseau	12½	the 8th of the hectolitre.

To find the weights:

1 gramme is about 15 grains, but rather exceeds it; 1 kilogramme, 2lb. 3oz. 5drs.

Problem.—*Find the weight of a given volume of water.*

The kilogramme is the weight of a cubic decimetre of very pure distilled water.—Take a regular formed vessel, that of a cylinder for example, such as drinking-glasses, bottles, &c.: measure the interior height and breadth to ascertain its capacity by calculation; fill the vessel with water, and you will obtain the required weight of the contained water, by taking a kilogramme for each cubic decimetre, a gramme for each cubic centimetre, &c.

For example: a cylindrical glass is 7,32 centimetres broad, to what height should it be filled to give the weight of a quarter of a kilogramme, that is to say, a quarter of a cubic decimetre, or

250 cubic centimetres. The superficies of the base is obtained by multiplying the radius 3,66 by 3,66 centimetres, and by $3\frac{1}{7}$; which gives 42 square centimetres. The height multiplied by 42, should give 250 cubic centimetres; dividing 250 by 42, the quotient 5,95 shews that you must put about the height of 6 centimetres of water in the glass, to obtain the weight of a quarter of a kilogramme, or 2 hectogrammes and a half.

Problem.—*Find the weight of a given volume of iron, copper, sand, &c.*

Calculate the weight of an equal volume of water, and multiply this number by the number affixed to the substance in question, in the underneath table.

Silver . .	10,70	Steel . . .	7,67	Sea-salt . .	1,92	Elm-wood .	0,67
Lead . .	11,35	Free-stone	2,42	Vil	0,91	Pear ditto .	0,66
Copper .	8,86	Marble . .	2,72	Lard or	0,95	Cherry ditto	0,72
Iron . . .	7,70	Chalk . . .	2,25	Butter		Wine . . .	0,99
Pewter .	7,67	Sugar . . .	1,61	Oak-wood	1,07	Brandy . .	0,86

Example.—It is required to know the weight of a cubic metre of marble? As the cubic metre contains 1000 cubic decimetres, of which each weighs 1 kilogramme; if it was question of a volume of water, it would weigh 1000 kilogrammes; this number multiplied by 2,72, which is affixed to marble, in the table, gives for the weight of a cubic metre of marble 2720 kilogrammes.

Example.—An axle-tree has been forged of a prism of iron, 9,5 centimetres by 6,1 angular base, and 18 decimetres in length, what is the weight? I multiply 9,5 by 6,1, and by 180, and I find for the volume of the iron 1043 cubic centimetres, or 10,431 cubic decimetres. The weight of an equal quantity of water is 10,431 kilogrammes. I multiply this number by 7,70, and find 80,32 kilogrammes for the weight of the axle-tree.

Example.—It is required to know the weight of the cask of which the gage was given (page 81) when filled with brandy? You have already ascertained its volume to be 257,641 cubic decimetres, if the vessel was full of water, it would therefore weigh 257,641 kilogrammes; this multiplied by 0,86, the number given in the table, you find 222 kilogrammes for the weight of brandy contained in the cask.

Example.—What is the weight of the loaf of sugar, of which we have found (page 81) that the contents is 6,177 cubic decimetres, equal to 6,177 kilogrammes of water? Multiply this number by 1,61, which will give 9,95 kilogrammes, the weight of the sugar.

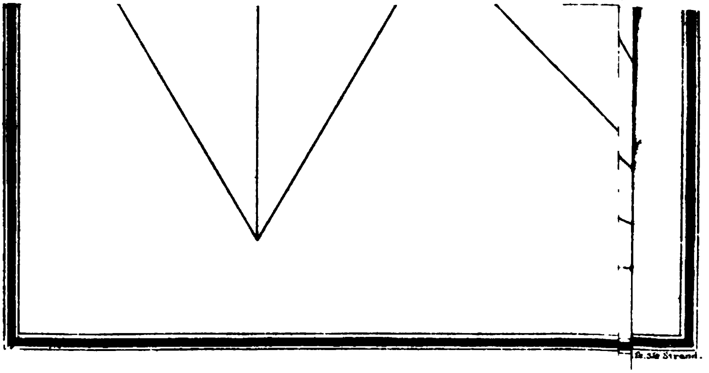
THE END.

Note.—All these calculations may be reduced to English measure, by calculating 2lb. 3oz. 5drs. avoirdupois weight for each kilogramme, or 5drs. avoirdupois for each decagramme, 3 feet four inches and a half for each metre, 2,064 gallons of wine measure, (not reckoning fractions) for every 10 litres.—*See Preface.*

MONITORS' TABLETS.

OBSERVATION.

These tablets should be detached from the work, pasted on separate pasteboards, and each given to the monitor of the class it refers to. The monitor reads from it the order he is to dictate to his class, and when the class is ranged in semicircles before the tablet against the wall, he must take care to point at the same time to the diagram to which his order refers.



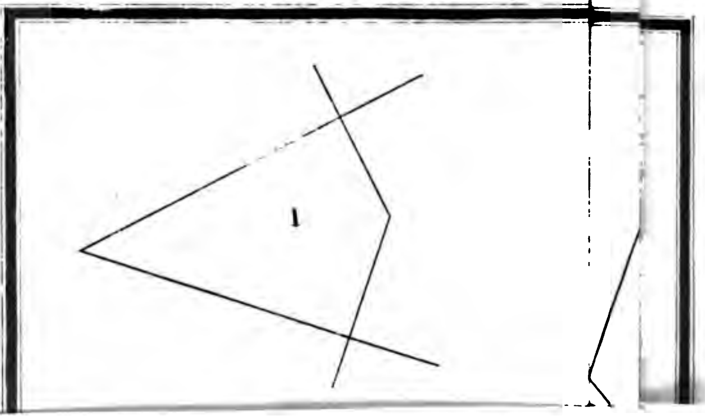
316 Strand.

MONITOR'S TABLET.

First Class—First Part.

1. Draw a right line, fig. 1.
2. Draw a right line, and divide it in the middle, fig. 2.
3. Draw a right line, and cut it in four equal parts, fig. 3.
4. Trace a right line, and prolong it of an equal length, fig. 2.
5. Trace a right line, prolong it of a double length, fig. 4.
6. Trace a right line, and prolong it of a triple length, fig. 3.
7. Cut a right line in three equal parts, fig. 4.
8. Cut a right line in 6 equal parts, or 8 equal parts.
9. Find the half of a right line, fig. 2.
10. Find the quarter and three quarters of a right line, fig. 3.
11. Find the third and two-thirds of a right line, fig. 4.
12. Draw a right line of 1, 2, 3, decimetres long, fig. 1.
13. Draw a right line, and divide it into decimetres, fig. 2 to 4.
14. What is the height and breadth of a boisseau of grain. See Tablet.
15. Draw a horizontal line, and divide it into 2, 3, 4, equal parts.*
16. Draw a vertical line, cut it in 2, 4, 3, equal parts, fig. 5.
17. Equidistant horizontal (or vertical) lines, fig. 1 to 14.
18. Trace oblique lines, and divide them in 2, 4, 3, equal parts, fig. 6.
19. Trace parallel and equidistant oblique lines, fig. 6.
20. Join two points given by a right line, fig. 1 to 6.
21. Trace a horizontal line from right to left, fig. 1 to 14.
22. Trace a vertical line from the bottom to the top, fig. 5.
23. From a given point draw a parallel to a right line, fig. 6.
24. Draw an acute angle, fig. 7; an obtuse angle, fig. 28.
25. Form an acute angle, whose opening shall be turned upwards, downwards, or to the left.
26. Form a triangle, fig. 11, 18, 19, 20.

* Repeat all the preceding questions, adding the condition that the lines should be horizontal, vertical, or oblique.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MONITOR'S TABLET.

FIRST CLASS—SECOND PART.

1. Draw a horizontal and a vertical line to cross each other, fig. 8.
2. Draw a perpendicular line upon a given right line, fig. 9.
3. Trace two lines which shall cross at right angles, fig. 9.
4. Trace a rectangle triangle, fig. 10 and 11.
5. Form an isosceles rectangle triangle, fig. 11.
6. Form a rectangle, fig. 12.
7. Form a rectangle, and divide it in equal rectangles, fig. 13.
8. Form a parallelogram, mark its altitude, fig. 14.
9. Form a square, fig. 15.
10. Trace angles with parallel sides, fig. 16 and 17.
11. Draw oblique lines equally diverging from the perpendicular, fig. 18.
12. Form an isosceles triangle, fig. 18; (give the base and altitude).
13. Trace a scalane triangle, then a second with parallel sides, fig. 19, 20.
14. Form an equilateral triangle, fig. 21; (give the side).
15. From a given point beyond a right line, draw a perpendicular to that right line, fig. 8 and 9.
16. Draw a perpendicular to a given point on a right line, fig. 8 and 9.
17. Draw a perpendicular on the end of a line, fig. 10.
18. Form a rhomboid or lozenge, fig. 22.
19. Form a square, with diagonals placed obliquely (or one being horizontal, the other vertical), fig. 23.
20. Form an equilateral triangle, of which the vertex shall be under the base, fig. 24.
21. Form an equilateral triangle placed obliquely,
22. Divide a right angle in half, fig. 25.
23. Divide an angle in two equal parts, fig. 26, (in four eight parts),
24. Double an angle, fig. 26.
25. Triple an angle, fig. 27.
26. Divide an angle in three equal parts, fig. 27,
27. Divide an obtuse angle in six equal parts.

36. Form a triangle, the three sides being given.

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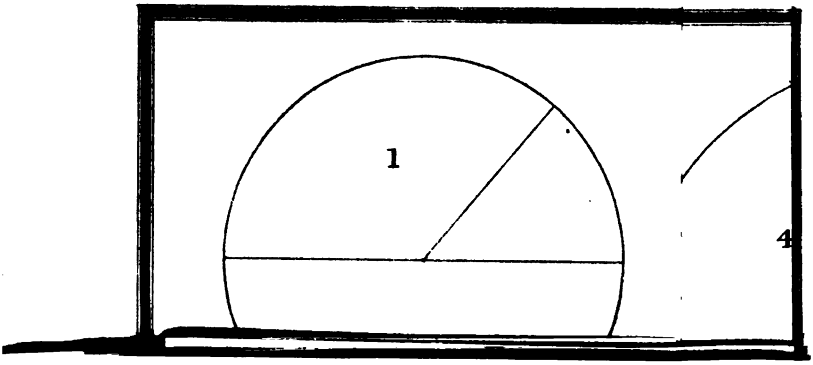

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MONITOR'S TABLET.

THIRD CLASS.

1. Trace a circle and mark its centre, a radius, and a diameter, fig. 1.
2. Form a circle, of which the centre and radius is given, fig. 1.
3. Divide a circle by two perpendicular diameters, fig. 3.
4. Divide a circle in eight equal parts, fig. 3.
5. Describe concentric circles (equidistant or not), fig. 2.
6. Trace two circles, the diameter of one being double or triple that of the other.
7. Trace an arc of a circle, and mark its centre and a radius, fig. 4 and 5.
8. Trace an arc to a given radius, fig. 4 and 5.
9. Divide an arc in half, fig. 4; in three parts, fig. 5.
10. Trace a circle, draw a tangent to a given point on the curve, fig. 6.
11. Trace an arc and a tangent on a given point of this arc, fig. 7.
12. Draw four tangents to a circle, forming a quadrilater, fig. 8.
13. Circumscribe a square on a circle, fig. 8.
14. Inscribe a square in a circle, fig. 9.
15. Double an arc, fig. 4; triple an arc, fig. 5.
16. Draw a tangent to a circle, from a given external point, fig. 6.
17. Draw two tangents to a circle from a given external point.
18. Divide a circle into six equal parts, form the inscribed hexagon, fig. 11.
19. Divide a circle into three equal parts, inscribe an equilateral triangle, fig. 12.
20. Form two unequal circles, tangent to each other, internally, fig. 14; externally, fig. 13.
21. The same; the centres and points of contact being given, fig. 13 and 14.
22. Inscribe a regular octagon in a circle, fig. 15.
23. Inscribe a regular pentagon in a circle, fig. 16.
24. Trace a regular polygon of 5, 6, or 8 sides, without forming a circle.
25. Form a triangle, and describe the circumscribed circle, fig. 12.
26. Form a circle, and trace a tangent triangle, fig. 18.
27. Trace a circle, which shall pass through all the vertices of a given regular polygon, fig. 9, 11, 12, 15, and 16.
28. Inscribe a circle in a triangle, fig. 19.
29. Form an arc which shall pass through two given points, fig. 4 and 7.
30. Form several arcs which shall pass through two given points, fig. 17.
31. Form a circle to pass through three given points, fig. 1.
32. Circumscribe a circle on a given triangle, fig. 12.
33. Circumscribe a polygon, regular or irregular, on a given circle, fig. 20.
34. Inscribe a circle within a given regular polygon, fig. 20.
35. Inscribe and circumscribe 2 parallel polygons to a circle, fig. 21.
36. Form a triangle, the three sides being given.

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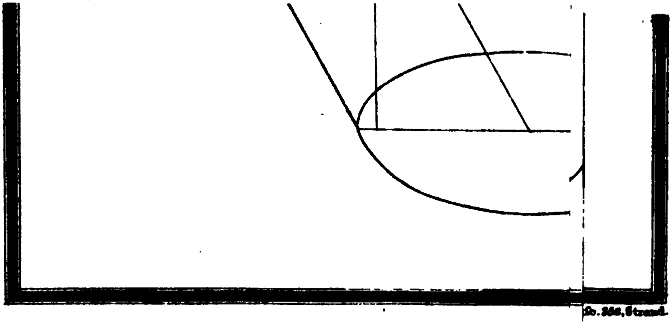

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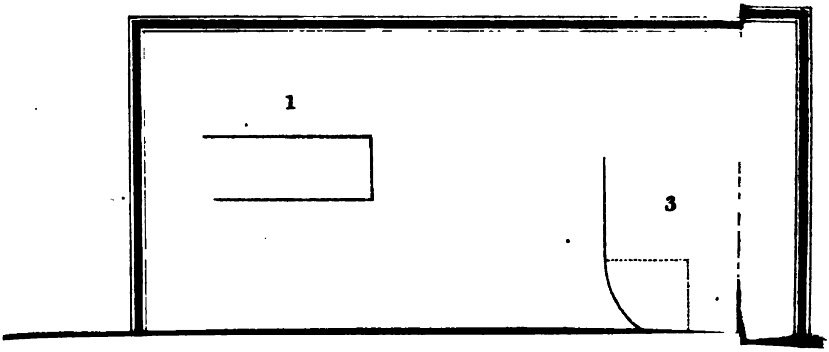
MONITOR'S TABLET.

FOURTH CLASS.

1. Draw a right line which shall touch two circles, fig. 1.
2. Describe four tangents to two circles which do not intersect each other, fig. 1.
3. Add two squares, fig. 2.
4. Double a square, fig. 3.
5. Subtract one square from another, fig. 2.
6. Take the half of a square, fig. 3.
7. Add three squares, fig. 4.
8. Triple a square, fig. 4.
9. Construct a protractor, fig. 5.
10. Form an angle of 36 degrees, of 50 degrees, &c., fig. 5.
11. Construct a sphere and its meridians, fig. 6.
12. Construct a sphere and its small circles which divide it into zones, fig. 7.
13. Draw an atlas, fig. 8.
14. Construct an ellipse, fig. 9 and 10.
15. Trace an oblique cone, fig. 11.
16. Draw a right cone, fig. 12.
17. Trace a right cylinder.
18. Draw an oblique cylinder.
19. Form a section parallel to the base of a cone or a cylinder, fig. 11, 12, 13, 14.
20. Double a cylinder or subtract a half; triple it or subtract a third, fig. 13 and 14.
21. Form a cylinder with a horizontal axis, fig. 15.



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MONITOR'S TABLET.

FIFTH CLASS.

1. Trace a filet, fig. 1.
2. Trace an astragal, fig. 2.
3. Trace an ovolo, fig. 3.
4. Draw a torus with its plinth, fig. 4.
5. Trace a quarter round with its filets, fig. 5.
6. Trace a reversed quarter round with its filets, fig. 6.
7. Draw a talon with its filets, fig. 7.
8. Draw a reversed talon with its filets, fig. 8.
9. Trace a cyma recta with its filets, fig. 9.
10. Trace a reversed cyma recta with its filets, fig. 10.
11. Draw all these mouldings in the opposite direction, facing them towards the left.
12. Draw fig. 11.
13. Draw fig. 12.
14. Draw fig. 13.
15. Draw fig. 14.
16. Draw fig. 15.
17. Draw fig. 16.
18. Draw fig. 17.
19. Draw fig. 18.

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